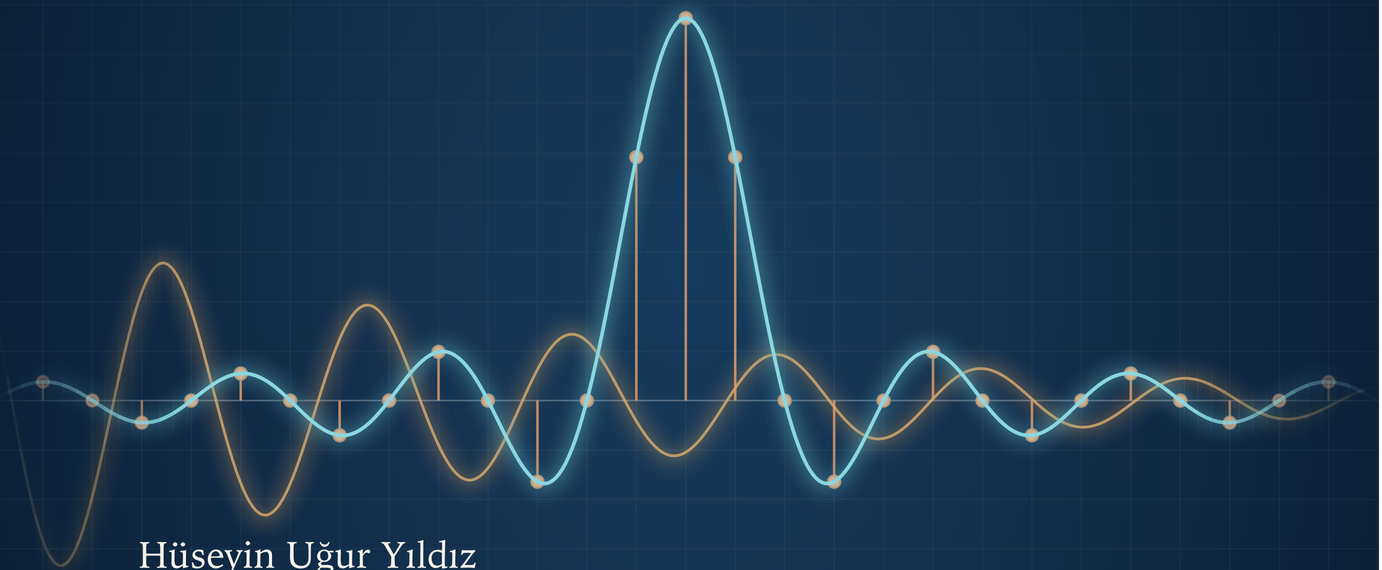
SIGNALS AND SYSTEMS

Signals, Systems and Frequency-Domain Analysis

Student Workbook



Hüseyin Uğur Yıldız
IEEE Senior Member
Associate Professor of Electrical and Electronics Engineering
TED University, Ankara, Türkiye

210 QUESTIONS · MODULES 1–7

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Contents

Every question in the course, with no answer and no solution. Work each one on the page, then check it against the artifact or against the instructor edition.

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30 questions

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How to use it

The questions are in the order the course meets them, and each has a stable question number. Only the statement and its lettered parts are printed; the reasoning stays for you to supply. The question numbers are shared with every other edition, so D5-04 is the same question in the artifact, in this workbook and in the instructor solutions.

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MODULE

1

Signal Foundations

30 questions on signal foundations. Write your reasoning in the space under each one.

D1-01

Decide whether each sequence below is periodic. Where it is, give the fundamental period N_0.

$$\text{(i)}\;\; x[n] = \cos\left(\tfrac{5\pi}{8}n - \tfrac{\pi}{6}\right) \qquad \text{(ii)}\;\; x[n] = e^{j3n} \qquad \text{(iii)}\;\; x[n] = \sin\left(\tfrac{\pi}{6}n\right)$$

a) Apply the periodicity test to each sequence.

b) Give N_0 wherever it exists, and say why it does not exist otherwise.

D1-02

Let

$$x[n] = \cos\left(\tfrac{\pi}{5}n\right) + \sin\left(\tfrac{\pi}{3}n\right).$$

a) Show that each term is periodic and give its period.

b) Determine the fundamental period N_0 of $x[n]$ and the fundamental frequency.

D1-03

Decide whether each continuous-time signal is periodic. Where it is, give T_0 and ω_0.

$$\text{(i)}\;\; x(t) = \cos(3t) + \sin(5t) \qquad \text{(ii)}\;\; y(t) = \cos(t) + \cos(\sqrt{2}\,t)$$

a) Give the period of each term separately.

b) Decide whether the sum repeats, and give T_0 where it does.

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D1-04

Let

$$x(t) = 2e^{-0.3t}\cos(4t), \qquad t \in \mathbb{R},$$

a sinusoid whose envelope is $\pm 2e^{-0.3t}$.

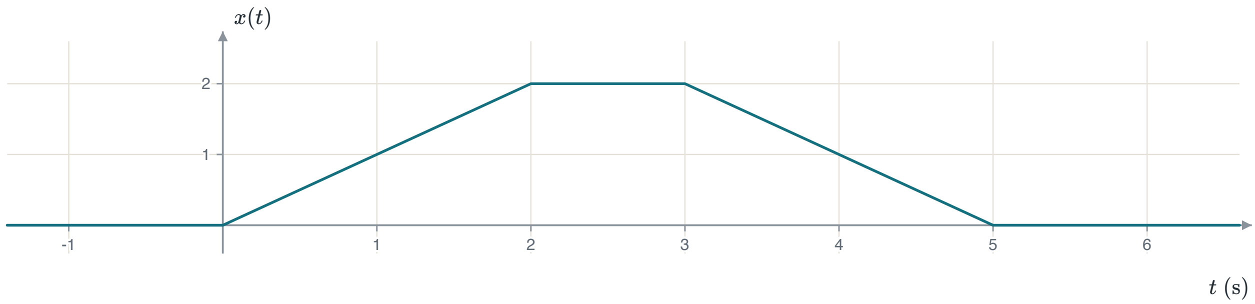
a) State the period T_0 of the pure oscillation $y(t) = 2\cos(4t)$, the case $r = 0$.

b) Prove that $x(t)$ itself is not periodic, by testing the necessary condition $x(0) = x(T)$ for a candidate period $T > 0$.

c) Explain why no signal whose envelope is strictly monotonic can be periodic.

D1-05

The signal $x(t)$ sketched below is zero outside $0 \le t \le 5$.



a) Calculate the total energy E_∞.

b) Calculate the average power P_∞ and classify the signal.

D1-06

Consider the two discrete-time signals

$$x_1[n] = \left(\tfrac{2}{5}\right)^n u[n], \qquad x_2[n] = 3\,u[n].$$

a) Calculate E_∞ and P_∞ for $x_1[n]$.

b) Calculate E_∞ and P_∞ for $x_2[n]$.

c) Classify each signal.

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D1-07

With the normalised convention $R = 1\,\Omega$, consider

$$x_1(t) = e^{-5t}u(t)\ \mathrm{V}, \qquad x_2(t) = 4\cos(3t)\ \mathrm{V}.$$

a) Calculate E_∞ for $x_1(t)$ and state P_∞.

b) Calculate P_∞ for $x_2(t)$ and state E_∞.

c) Starting from $P_T = \dfrac{1}{2T}\displaystyle\int_{-T}^{T} |x(t)|^2\,\mathrm{d}t$, show directly that a finite E_∞ forces $P_\infty = 0$.

D1-08

Let

$$x[n] = \begin{cases} 2^{\,n}, & n \le -1, \\ 3, & n = 0, \\ \left(\frac{1}{2}\right)^n, & n \ge 1. \end{cases}$$

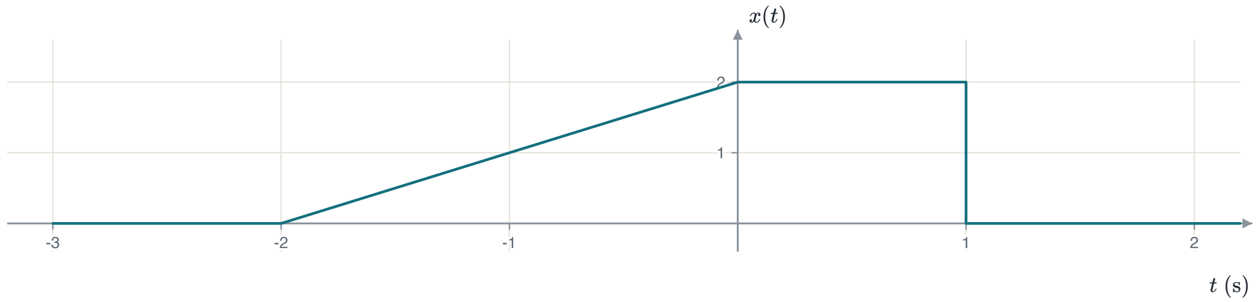
a) List $x[n]$ for $-3 \le n \le 3$.

b) Calculate E_∞.

c) State P_∞ and classify the signal.

D1-09

The signal $x(t)$ is sketched below: it rises linearly from 0 at $t = -2$ to 2 at $t = 0$, stays at 2 until $t = 1$, and is zero elsewhere.

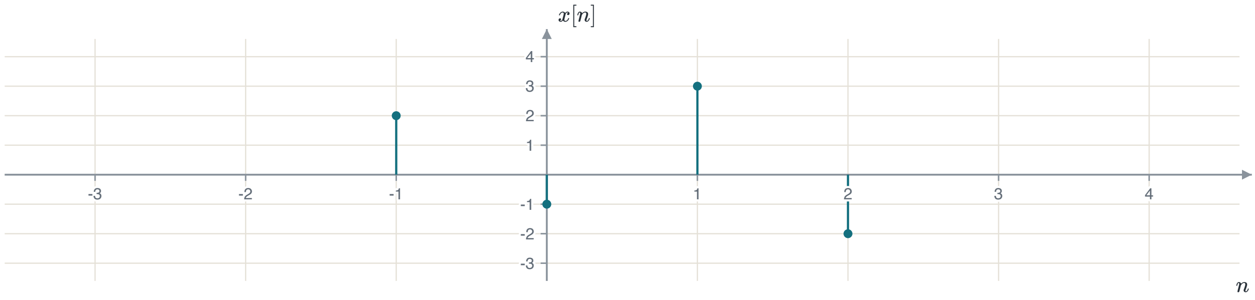


a) Plot $y(t) = x(2t + 2)$.

b) State the support of $y(t)$ and check its width against the width of $x(t)$.

D1-10

The sequence $x[n]$ is plotted below.

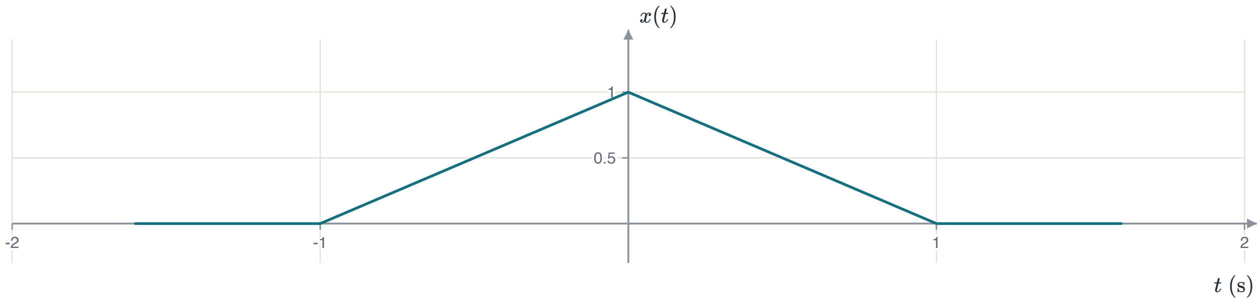


a) Plot $y[n] = x[2 - n]$.

b) Plot $z[n] = x[n + 1]$ and say which of the two operations changes the order of the samples.

D1-11

The signal $x(t)$ sketched below is a symmetric triangular pulse of height 1, zero outside $-1 \leq t \leq 1$.



a) Plot $y(t) = x\left(1 - \frac{t}{3}\right)$.

b) State the support of $y(t)$ and check its width against the width of $x(t)$.

c) Verify the location of the peak of $y(t)$ by direct substitution.

D1-12

A signal $x(t)$ is known only to satisfy $x(t) = 0$ outside $2 \leq t \leq 8$, with a single, unique maximum at $t = 5$. No other property of $x(t)$ is given. Let $y(t) = x(-3t + 6)$.

a) Find the support of $y(t)$ as an interval, using inequalities on the argument of x.

b) Find the value of t at which $y(t)$ reaches its maximum.

c) A second transformation is applied, $z(t) = y(t - 4)$. Find the support of $z(t)$ from the support of $y(t)$. Then find it again from the support of $x(t)$ and compare the results.

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D1-13

Let $x(t) = 3e^{-4t}u(t)$.

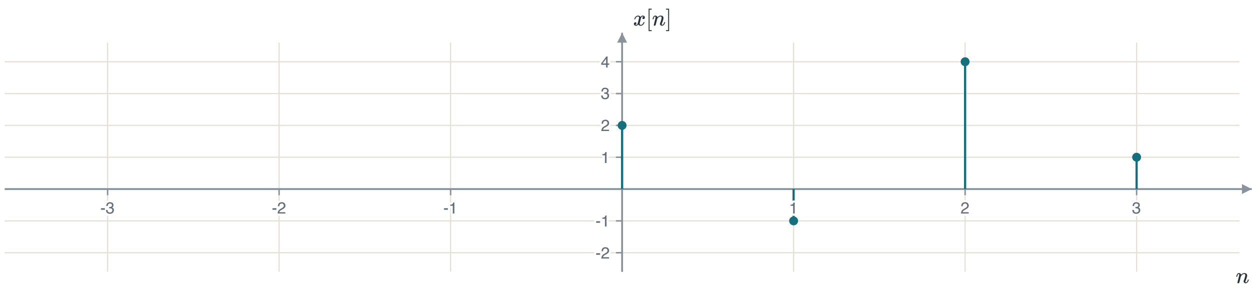
a) Find $\mathcal{E}\mathrm{v}\{x(t)\}$ and $\mathcal{O}\mathrm{dd}\{x(t)\}$ and plot both.

b) Calculate E_∞ for $x(t)$, for its even part, and for its odd part.

c) Comment on the relation between the three energies.

D1-14

The sequence $x[n]$ equals 2, -1, 4, 1 at $n = 0, 1, 2, 3$ and is zero elsewhere.



a) Plot $x[-n]$.

b) Plot $\mathcal{E}\mathrm{v}\{x[n]\}$ and $\mathcal{O}\mathrm{dd}\{x[n]\}$.

c) Verify that the two parts add back to $x[n]$ at $n = -1$, $n = 0$ and $n = 2$.

D1-15

Let

$$x(t) = \begin{cases} -1, & -2 \le t < 0, \\ 2, & 0 \le t \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

a) Determine $x(-t)$ as a piecewise formula.

b) Determine $\mathcal{E}\mathrm{v}\{x(t)\}$ and $\mathcal{O}\mathrm{dd}\{x(t)\}$, each as a piecewise formula.

c) Calculate E_x, $E_{\mathcal{E}\mathrm{v}}$ and $E_{\mathcal{O}\mathrm{dd}}$, and verify that the two parts' energies add to E_x.

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D1–16

Let $x[n]$ be an arbitrary real-valued sequence, not given by any formula.

a) Prove that $\mathcal{O}\mathrm{dd}\{x\}[0] = 0$, directly from the definition of the odd part.

b) The sequence $w[n]$ has $w[-1] = -5$, $w[0] = 0$, $w[1] = 5$, zero elsewhere. A student claims $\mathcal{O}\mathrm{dd}\{w\}[n] = w[n]$ for every n and $\mathcal{E}\mathrm{v}\{w\}[n] = 0$ for every n. Determine whether the claim is correct.

c) A second sequence has $y[0] = 4$. Using part (a) only, and without computing $\mathcal{O}\mathrm{dd}\{y\}$, explain why $y[n]$ cannot be an odd sequence.

D1–17

Evaluate each of the following.

$$\text{(i)} \int_{-\infty}^{\infty} (3t^2+1)\,\big[\delta(t+2)+\delta(t-4)\big]\,\mathrm{d}t \qquad \text{(ii)} \int_{-\infty}^{\infty} e^{-2t}\sin(\pi t)\,\delta(t-1.5)\,\mathrm{d}t \qquad \text{(iii)} \int_{-\infty}^{\infty} t^2\,\delta(4t-8)\,\mathrm{d}t$$

a) Evaluate the three integrals.

b) State which property of the impulse each one uses, and say why the answer is a number rather than a signal.

D1–18

Let

$$g(t) = 3\delta(t+1) - 2\delta(t-2), \qquad v(t) = \int_{-\infty}^{t} g(\tau)\,\mathrm{d}\tau.$$

a) Using $u(t) = \displaystyle\int_{-\infty}^{t} \delta(\tau)\,\mathrm{d}\tau$, express $v(t)$ as a combination of unit step functions.

b) Sketch $v(t)$ for $-3 \le t \le 4$.

c) Read $v(0)$ and $v(3)$ from the sketch, and check them against the formula from part (a).

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D1-19

Let $x[n]$ have $x[-1] = 5$, $x[0] = -3$, $x[1] = 2$, $x[2] = 7$, zero elsewhere.

a) Write the sampling-property result $x[n]\delta[n-1]$ explicitly as a sequence.

b) Write the sifting-property result $\sum_{n=-\infty}^{\infty} x[n]\delta[n-1]$ as a number, and confirm it equals $x[1]$.

c) Using the representation property $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$, write out its four non-zero terms for this $x[n]$, and confirm the sum reproduces $x[0]$.

D1-20

Let

$$x[n] = \delta[n] + 2\delta[n-2] - \delta[n-4], \qquad y[n] = \sum_{k=-\infty}^{n} x[k].$$

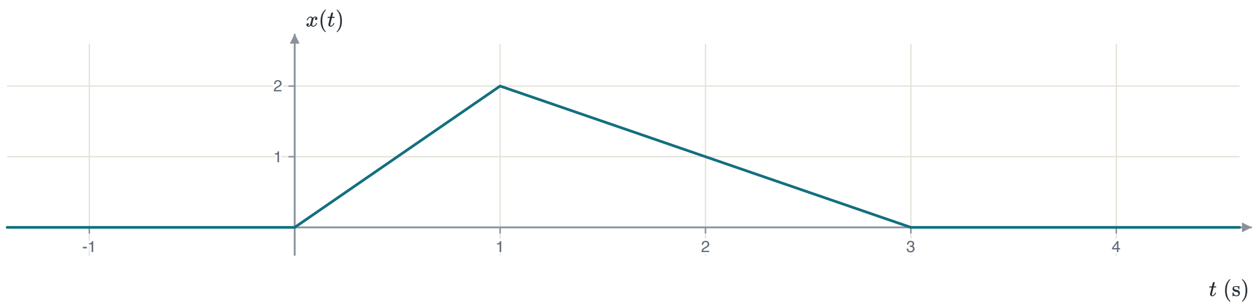
a) Evaluate $y[n]$ for $-2 \leq n \leq 6$.

b) Express $y[n]$ in closed form as a combination of unit step functions, and check it against part (a).

c) A continuous-time signal $p(t) = \delta(t) + 2\delta(t-2) - \delta(t-4)$ carries the same weights at the same locations. State its running integral $q(t) = \int_{-\infty}^{t} p(\tau)\,\mathrm{d}\tau$ directly by analogy with $y[n]$, without repeating the calculation.

D1-21

Work the following parts in order. Parts (b) to (e) all use the signal $x(t)$ plotted below.



a) Is $x[n] = \cos\left(\frac{4\pi}{9}n - 2\right)$ periodic? If so, find its fundamental period.

b) Calculate the energy E_∞ of $x(t)$.

c) Plot $y(t) = x\left(3 - \frac{t}{2}\right)$.

d) Plot $z(t) = \mathcal{E}\mathrm{v}\{y(t)\}$, the even part of the $y(t)$ signal of part (c).

e) Evaluate $\int_{-\infty}^{\infty} z(t)\{\delta(t+4) + \delta(t-8)\}\,\mathrm{d}t$, with $z(t)$ as defined in part (d).

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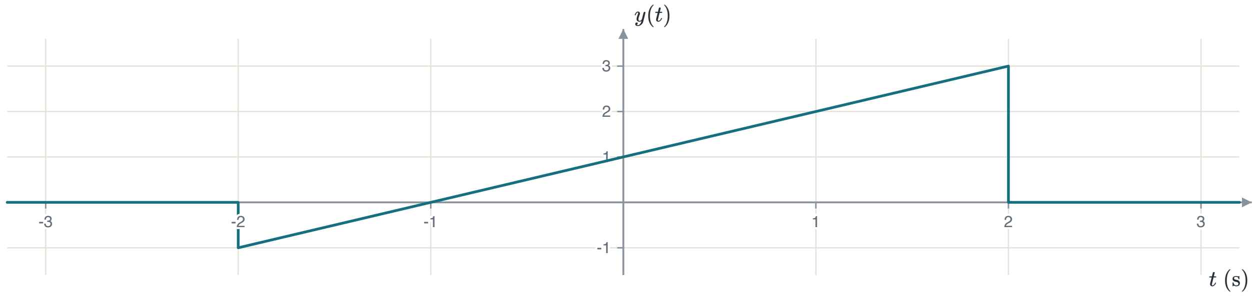
D1-22

Work the following parts in order. Parts (c) and (d) use the signal $y(t)$ plotted below.



a) Is the signal $x(t) = 2je^{j5t}$ periodic? If so, what is its fundamental period?

b) Determine the energy E_∞ of the signal $x[n] = 3^{-n}u[n]$.

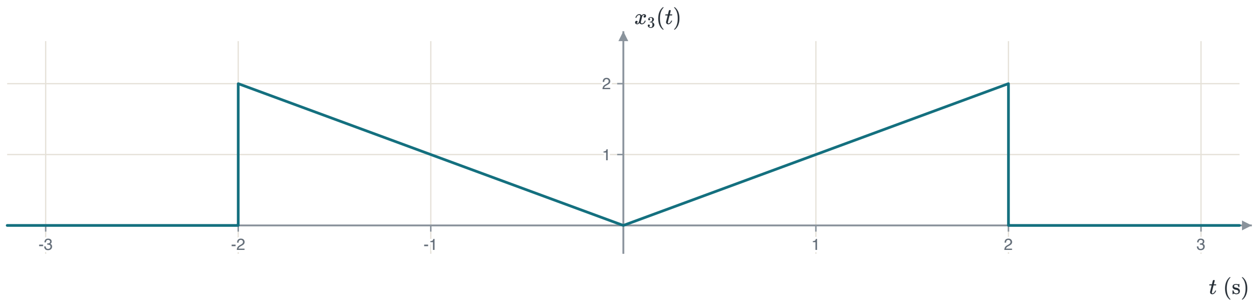
c) Generate a plot of $y(-3t+2)$ using the given signal $y(t)$.

d) Plot the even part of $y(t)$.

e) Evaluate $\int_{-\infty}^{\infty} e^{-t}\delta(2t-4)\,\mathrm{d}t$.

D1-23

Work the following parts in order. Parts (b) and (c) use the sequence $x_1[n]$ of part (a); parts (d) and (e) use the signal $x_3(t)$ plotted below.



a) Sketch and label the signal $x_1[n] = \sum_{k=-\infty}^{\infty}\{\delta[n-3k] - \delta[n+1+4k]\}$.

b) Is the $x_1[n]$ of part (a) periodic? If so, what is its fundamental period?

c) For the $x_1[n]$ of part (a), plot the odd part of $x_2[n] = \begin{cases} x_1[n], & -3 \le n \le 3 \\ 0, & \text{otherwise.} \end{cases}$

d) Calculate the energy E_∞ of $x_3(t)$.

e) For the $x_3(t)$ of part (d), evaluate $\int_{-\infty}^{\infty} x_3(t)\delta(t+0.6)\,\mathrm{d}t$.

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D1–24

Consider the signal

$$x(t) = [t + 3\{u(t+2) - u(t-2)\}] \times \{u(t+3) - u(t-3)\}.$$

a) Plot the even part of $x(t)$.

b) Plot the odd part of $x(t)$.

c) Calculate the energies of the even and odd parts of $x(t)$, and of $x(t)$ itself.

D1–25

Two discrete-time complex exponentials differ only in whether π appears in the frequency:

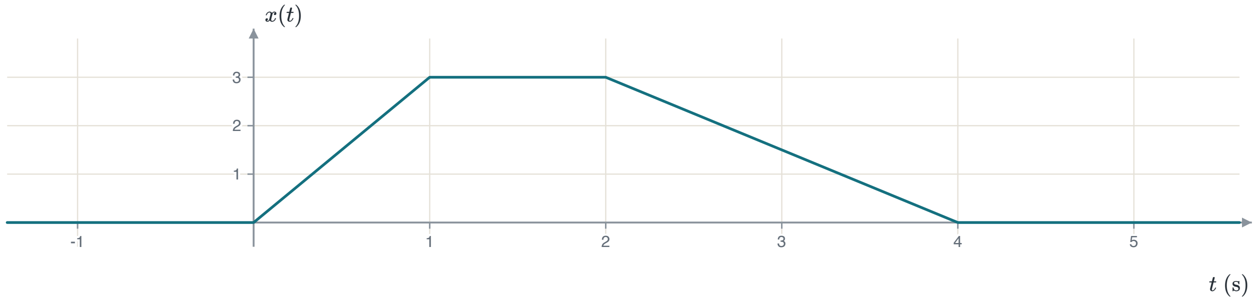
$$x_1[n] = e^{j\frac{4}{7}n}, \qquad x_2[n] = e^{j\frac{4\pi}{7}n}.$$

a) Determine whether $x_1[n]$ is periodic. If it is, find its fundamental period.

b) Determine whether $x_2[n]$ is periodic. If it is, find its fundamental period, and say what makes the two cases differ.

D1–26

Work the following parts in order. Parts (b) to (e) all use the signal $x(t)$ plotted below.



a) Is $x[n] = \sin\left(\frac{6\pi}{7}n + \frac{\pi}{4}\right)$ periodic? If so, find its fundamental period.

b) Calculate the energy E_∞ of $x(t)$.

c) Plot $y(t) = x(2t+4)$.

d) Plot $z(t) = \mathcal{O}\mathrm{dd}\{y(t)\}$, the odd part of the $y(t)$ signal of part (c).

e) Evaluate $\int_{-\infty}^{\infty} z(t)\delta(3t+3)\,\mathrm{d}t$, with $z(t)$ as defined in part (d).

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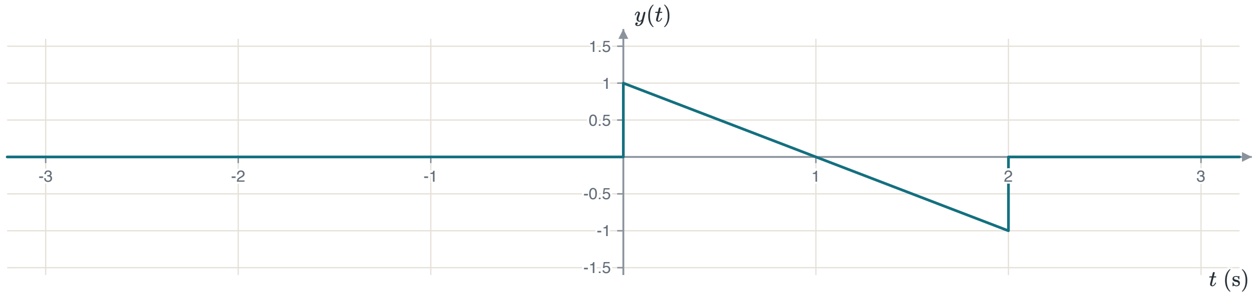
D1-27

Work the following parts in order. Parts (c) to (e) use the signal $y(t)$ plotted below.



a) Is the signal $x(t) = 5e^{j\frac{3\pi}{4}t}$ periodic? If so, what is its fundamental period?

b) Determine the energy E_∞ of the signal $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$.

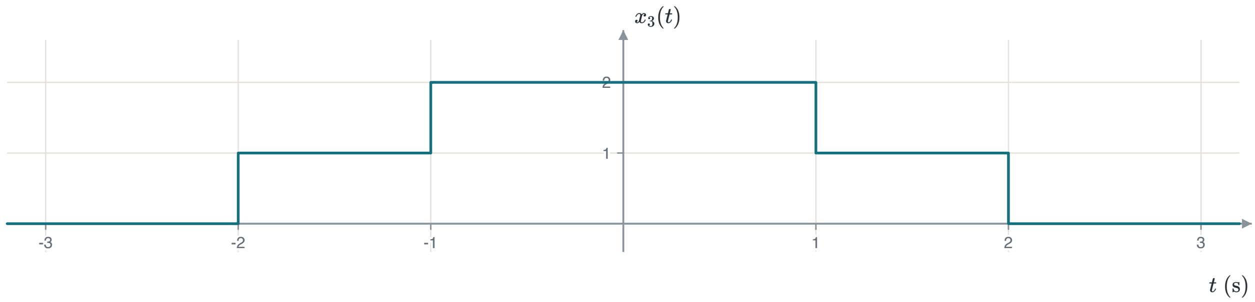
c) Generate a plot of $y(2t-1)$ using the given signal $y(t)$.

d) Plot the even part of $y(t)$.

e) Evaluate $\int_{-\infty}^{\infty} y(t)\delta(2t-1)\,\mathrm{d}t$.

D1-28

Work the following parts in order. Parts (b) and (c) use the sequence $x_1[n]$ of part (a); parts (d) and (e) use the signal $x_3(t)$ plotted below.



a) Sketch and label the signal $x_1[n] = \sum_{k=-\infty}^{\infty}\{\delta[n-4k] - \delta[n+1+6k]\}$.

b) Is the $x_1[n]$ of part (a) periodic? If so, what is its fundamental period?

c) For the $x_1[n]$ of part (a), plot the odd part of $x_2[n] = \begin{cases} x_1[n], & -4 \le n \le 4 \\ 0, & \text{otherwise.} \end{cases}$

d) Calculate the energy E_∞ of $x_3(t)$.

e) For the $x_3(t)$ of part (d), evaluate $\int_{-\infty}^{\infty} x_3(t)\delta(t-1.5)\,\mathrm{d}t$.

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D1-29

Consider the signal

$$x(t) = [2t + \{u(t+1) - u(t-1)\}] \times \{u(t+2) - u(t-2)\}.$$

a) Plot the even part of $x(t)$.

b) Plot the odd part of $x(t)$.

c) Calculate the energies of the even and odd parts of $x(t)$, and of $x(t)$ itself.

D1-30

A signal is built from three step functions:

$$x(t) = u(t+2) - 2u(t) + u(t-2).$$

a) Sketch $x(t)$ and give its piecewise form.

b) Determine $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ as a sum of impulses.

c) Evaluate $\int_{-\infty}^{\infty} x(t)\delta(t-1)\,\mathrm{d}t$.

d) Evaluate $\int_{-\infty}^{\infty} \dfrac{\mathrm{d}x}{\mathrm{d}t}\, t^2\,\mathrm{d}t$.

e) Calculate the energy E_∞ of $x(t)$.

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MODULE

2

Systems and Their Properties

30 questions on systems and their properties. Write your reasoning in the space under each one.

D2-01

Consider the system

$$y(t) = \cos\big(x(t)\big).$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer. An unjustified answer receives no credit.

D2-02

Consider the system

$$y[n] = x[n]\,x[n+1].$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2-03

Consider the system

$$y(t) = x^3(t) - x(t).$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

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D2-04

Two memoryless systems:

$$\text{(i)}\;\; y(t) = x^3(t), \qquad \text{(ii)}\;\; z[n] = x[n]\,x[n-2].$$

a) Determine whether system (i) is invertible. Give the inverse relation if it is.

b) Determine whether system (ii) is invertible. Give an explicit counterexample if it is not.

c) Explain why raising the input to an odd power preserves invertibility while forming a product of two different samples generally does not.

D2-05

Consider the system

$$y[n] = \sum_{k=n-3}^{n} x[k].$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2-06

Consider the system

$$y(t) = \int_0^t x(\tau)\,d\tau.$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2-07

Consider the system

$$y[n] = \sum_{k=-\infty}^{n} \left(\tfrac{1}{2}\right)^{n-k} x[k].$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

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D2-08

The running sum

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

is unstable in general. A particular input is

$$x[n] = \left(\tfrac{1}{3}\right)^n u[n].$$

a) Verify that $x[n]$ is bounded, and find a closed form for $y[n]$ valid for $n \geq 0$.

b) Show that this particular response is itself a bounded sequence.

c) Explain why part (b) does not establish that the system is BIBO stable, and produce a single bounded input whose response is unbounded.

d) State, in one sentence, the general distinction between a property of a system and a property of one of its responses.

D2-09

Consider the system

$$y(t) = e^{-t} x(t).$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2-10

Consider the system

$$y[n] = 2^{-|n|} x[n].$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

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D2-11

Consider the system

$$y(t) = x(t)\cos(\pi t) + x(t-1).$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2-12

Two systems built from a time-varying gain:

$$\text{(i)}\;\; y(t) = (t^2+1)\,x(t), \qquad \text{(ii)}\;\; y(t) = t\,x(t).$$

a) Determine whether system (i) is invertible. Give the inverse relation if it is.

b) Determine whether system (ii) is invertible. Give an explicit counterexample if it is not.

c) State the general condition on a gain $g(t)$, in the relation $y(t) = g(t)x(t)$, for the system to be invertible.

D2-13

Consider the system

$$y[n] = x[n+2].$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2-14

Consider the system

$$y(t) = x(3-t).$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

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D2–15

Consider the system

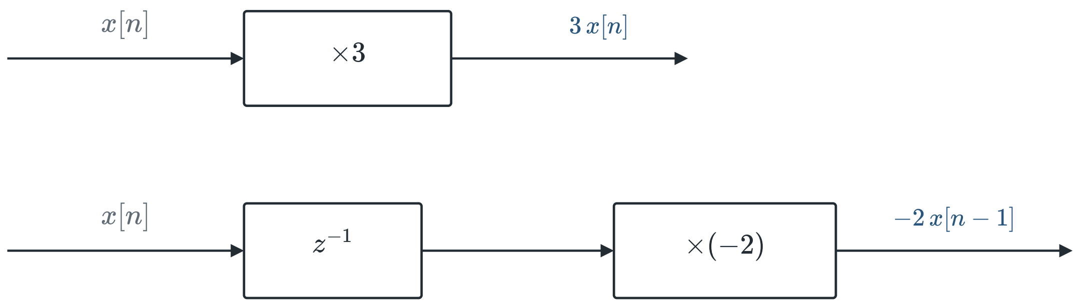
$$y[n] = x[3n-1].$$

a) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

b) Justify each answer.

D2–16

The block diagram below shows a system built from a one-sample delay and two fixed gains. The direct branch multiplies the input by 3; the delayed branch multiplies the once-delayed input by -2; the two branches are added.



a) Read the diagram and write the input–output relation it implements.

b) Determine whether the system is memoryless, linear, time invariant, causal and BIBO stable.

c) State the condition for the system to be invertible, and give the inverse relation.

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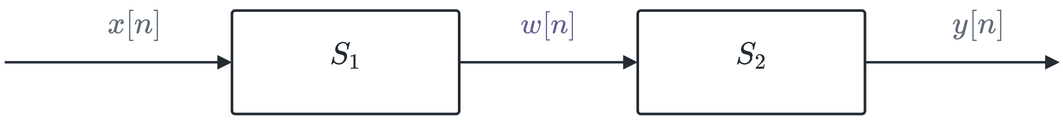
D2–17

Two systems are connected in series, the output of S_1 feeding directly into S_2. S_1 is the first difference, $w[n] = x[n] - x[n-1]$; S_2 is a fixed gain, $y[n] = 2w[n]$.



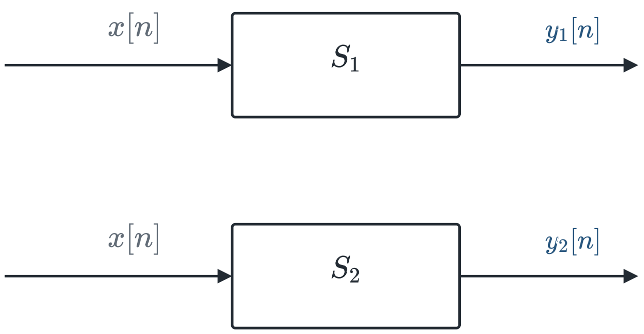
a) Write the overall input–output relation of the series connection, $y[n]$ in terms of $x[n]$ alone.

b) S_1 and S_2 are each linear and each time invariant. Argue directly from the definitions that a series connection of two linear systems is linear, and that a series connection of two time-invariant systems is time invariant.

c) Verify both conclusions directly from the relation found in part (a).

D2–18

Two systems are connected in parallel, both acting on the same input and their outputs added: S_1 is the identity, $y_1[n] = x[n]$; S_2 is a unit delay, $y_2[n] = x[n-1]$.



a) Write the overall input–output relation of the parallel connection.

b) S_1 and S_2 are each causal and each BIBO stable. Argue directly from the definitions that a parallel connection of two causal systems is causal, and that a parallel connection of two BIBO-stable systems is BIBO stable.

c) Confirm both conclusions directly for this particular combination.

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D2-19

The system below is built with feedback:

$$y[n] = x[n] - a\,y[n-1],$$

where a is a real constant, the output returning through a gain a and a one-sample delay and entering with a negative sign.

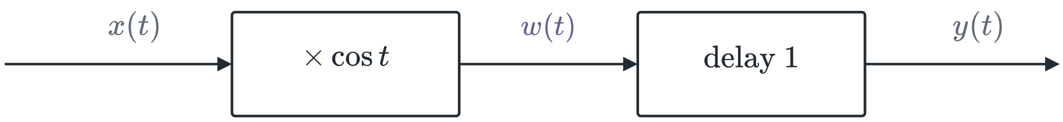
a) Determine whether the system is memoryless, causal and linear, for any real value of a.

b) Iterate the recursion to express $y[n]$ as a sum involving only $x[n], x[n-1], x[n-2], \ldots$, assuming $y[k] \to 0$ as $k \to -\infty$.

c) Using the sum from part (b), state the condition on a under which the system is BIBO stable, and justify it.

D2-20

Two systems are connected in series: S_1 is a modulator, $w(t) = x(t)\cos(t)$; S_2 is a fixed one-second delay, $y(t) = w(t-1)$.



a) Determine whether S_1, taken alone, is time invariant.

b) Determine whether S_2, taken alone as a system with input w and output y, is time invariant.

c) Write the overall relation for the series connection, and determine whether it is time invariant. Justify with a counterexample if it fails.

d) Determine whether the overall connection is linear, using the fact that a series connection of two linear systems is linear.

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D2-21

Consider the following input and output relationship of a system

$$y[n] = \sum_{k=0}^{n+2} \sin\left(\tfrac{\pi}{6}k\right) x[k].$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer; an unjustified answer earns nothing.

D2-22

Consider the following input-output relationship of a system.

$$y(t) = (t+2)\, x(t-3)$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer.

D2-23

Consider the following input and output relationship of a system

$$y(t) = \mathcal{O}\mathrm{dd}\{x(t)\} = \tfrac{1}{2}[x(t) - x(-t)].$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer.

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D2–24

Consider the following input and output relationship of a system

$$y[n] = x[n]\,x[n-2].$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer.

D2–25

Consider the following input-output relationship of a system.

$$y(t) = \int_{-\infty}^{2t} x(\tau)\,\mathrm{d}\tau$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer.

D2–26

Consider the following input and output relationship of a system

$$y[n] = n\,x[n+1].$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer.

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D2-27

Consider the following input-output relationship of a system.

$$y(t) = e^{x(t)}$$

a) Determine whether the system is memoryless.
b) Determine whether the system is linear.
c) Determine whether the system is time-invariant.
d) Determine whether the system is causal.
e) Determine whether the system is stable. Justify every answer.

D2-28

Consider the following input and output relationship of a system

$$y[n] = \sum_{k=n-3}^{n+3} x[k].$$

a) Determine whether the system is memoryless.
b) Determine whether the system is linear.
c) Determine whether the system is time-invariant.
d) Determine whether the system is causal.
e) Determine whether the system is stable. Justify every answer.

D2-29

Consider the following input-output relationship of a system.

$$y(t) = x\left(\frac{t}{3}\right)$$

a) Determine whether the system is memoryless.
b) Determine whether the system is linear.
c) Determine whether the system is time-invariant.
d) Determine whether the system is causal.
e) Determine whether the system is stable. Justify every answer.

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D2-30

Consider the following input and output relationship of a system

$$y(t) = x(t)\cos(3t).$$

a) Determine whether the system is memoryless.

b) Determine whether the system is linear.

c) Determine whether the system is time-invariant.

d) Determine whether the system is causal.

e) Determine whether the system is stable. Justify every answer.

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MODULE

3

Linear Time-Invariant Systems

30 questions on linear time-invariant systems. Write your reasoning in the space under each one.

D3–01

The input and output of a discrete-time LTI system are related by

$$y[n] = 2x[n] - x[n-1] + 3x[n-3].$$

a) Determine and plot the impulse response $h[n]$.

b) For the input $x[n] = u[n] - u[n-3]$, compute and plot $y[n] = x[n] * h[n]$.

D3–02

A discrete-time LTI system is described by

$$y[n] = \frac{1}{4}\, y[n-1] + x[n],$$

and is initially at rest.

a) Determine and plot the impulse response $h[n]$.

b) For the input $x[n] = u[n]$, compute and plot $y[n]$.

c) State the limit of $y[n]$ as $n \to \infty$ and say what it means.

D3–03

A discrete-time LTI system is described by

$$y[n] = x[n+1] - 2x[n] + x[n-1],$$

which uses a future value of the input and is therefore not causal.

a) Determine and plot the impulse response $h[n]$.

b) For the input $x[n] = u[n+1] - u[n-2]$ (equal to 1 at $n = -1, 0, 1$), compute and plot $y[n] = x[n] * h[n]$.

c) Using only $\sum_n h[n]$ and $\sum_n x[n]$, state $\sum_n y[n]$ without adding the samples of part (b), and confirm the two agree.

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D3-04

Two discrete-time LTI systems are described by their own difference equations,

$$y_1[n] = x[n] + x[n-1] \qquad \text{and} \qquad y_2[n] = x[n] - x[n-1].$$

a) Determine $h_1[n]$ and $h_2[n]$.

b) The two systems are placed in cascade. Determine and plot the impulse response of the cascade, $h_c[n] = h_1[n] * h_2[n]$.

c) The same two systems are placed in parallel instead, outputs summed. Determine $h_p[n] = h_1[n] + h_2[n]$, and use it to find the response to $x[n] = 3\delta[n-4]$.

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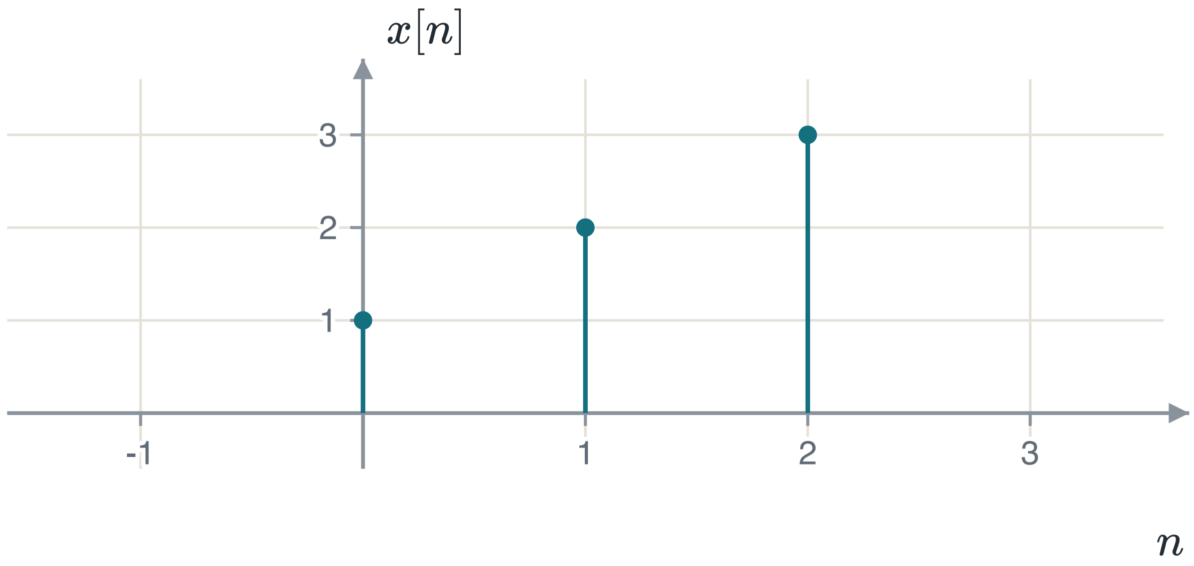
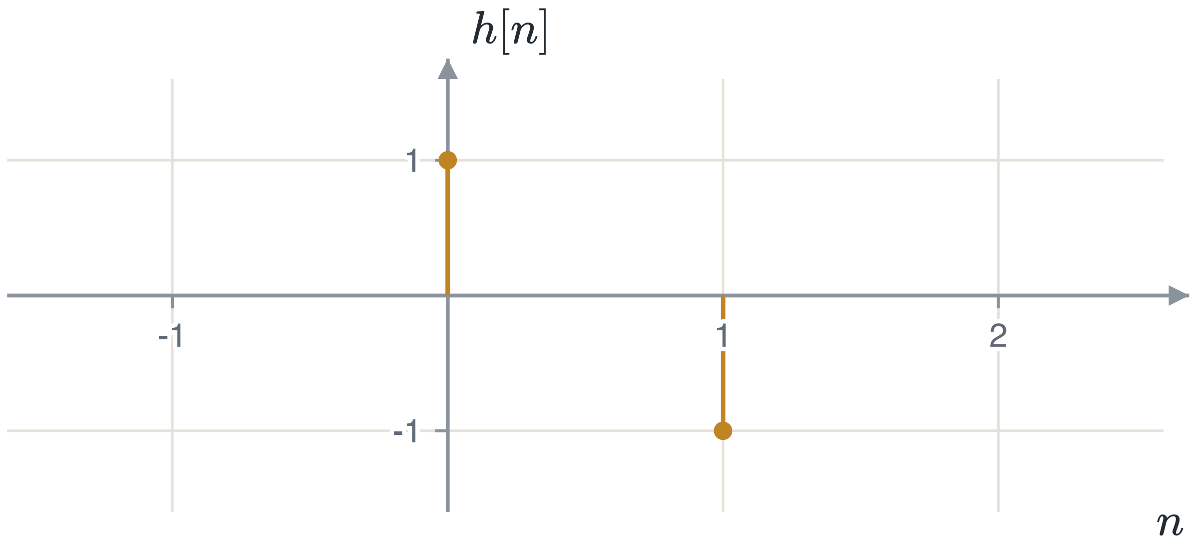
D3-05

The sequences $x[n]$ and $h[n]$ of a discrete-time LTI system are shown below.



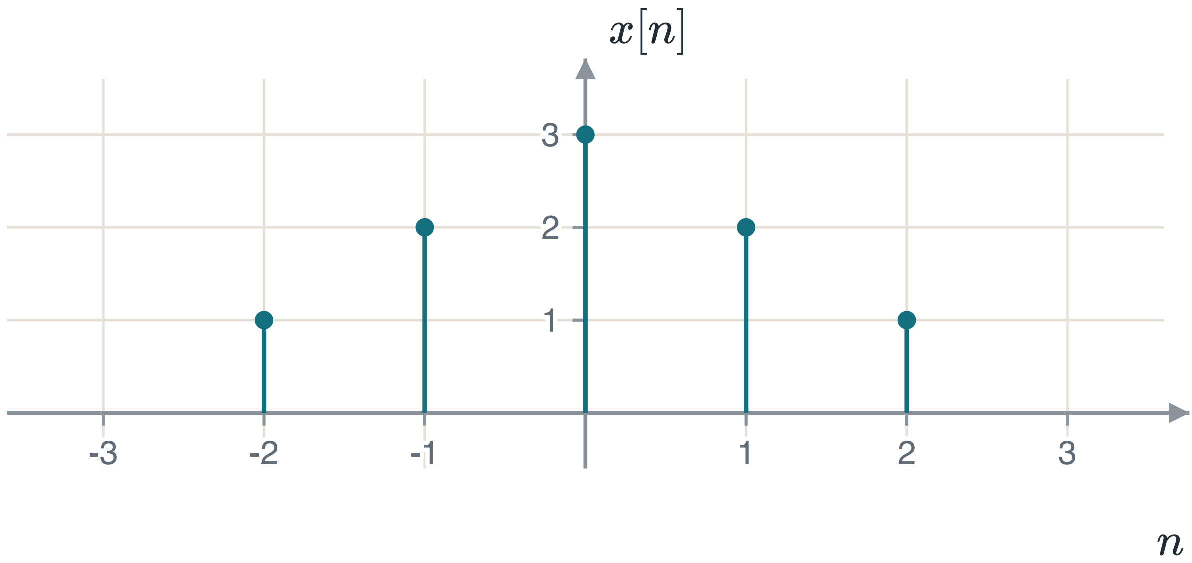


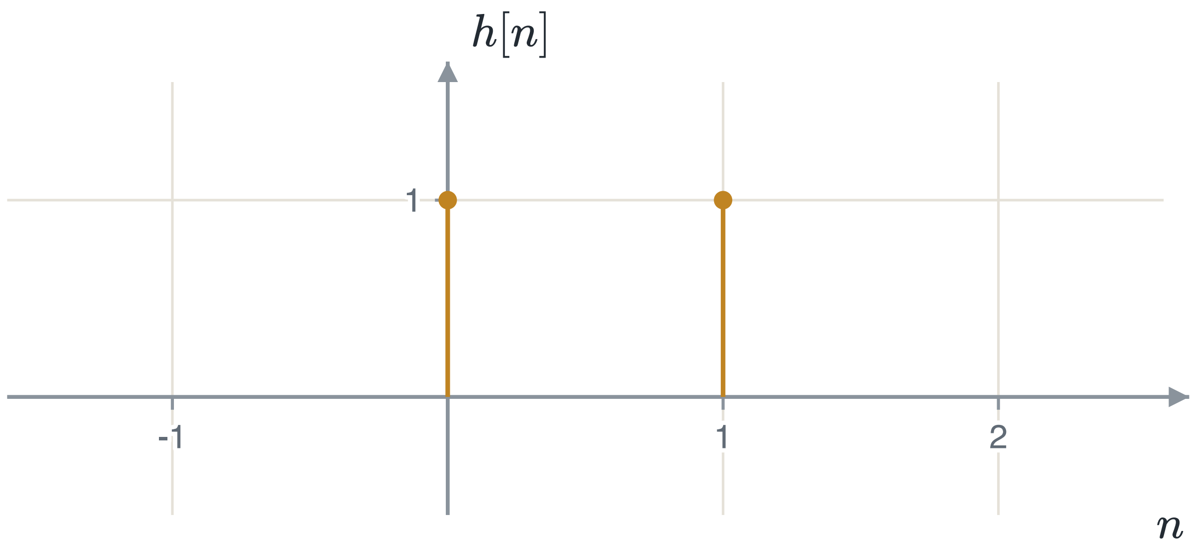
a) Compute and plot $y[n] = x[n] * h[n]$.

b) State which sample of $y[n]$ has the largest magnitude, and explain why.

D3-06

The sequences $x[n]$ and $h[n]$ of a discrete-time LTI system are shown below.



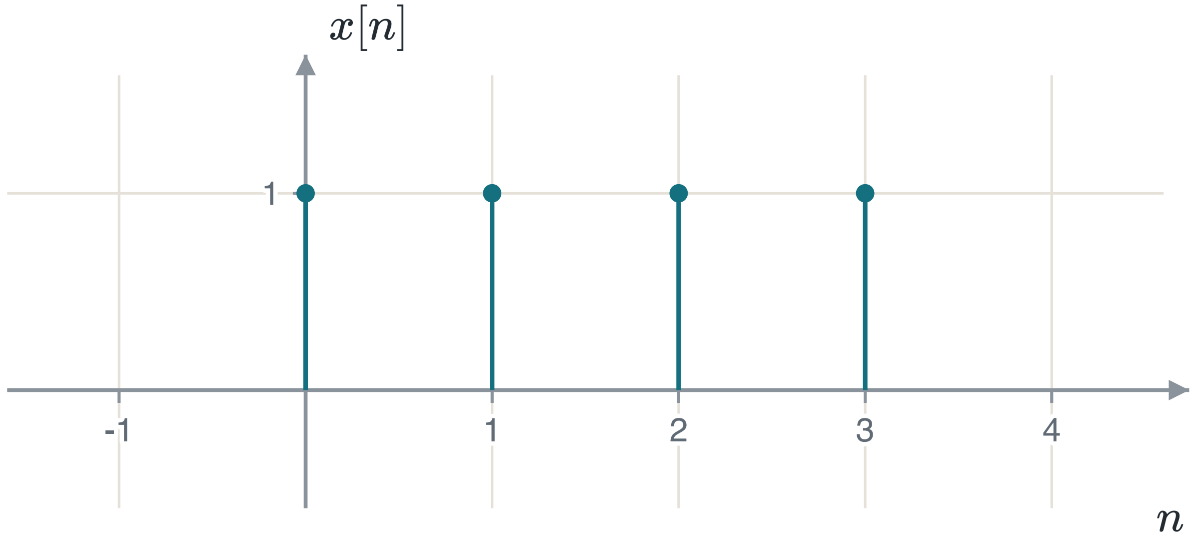


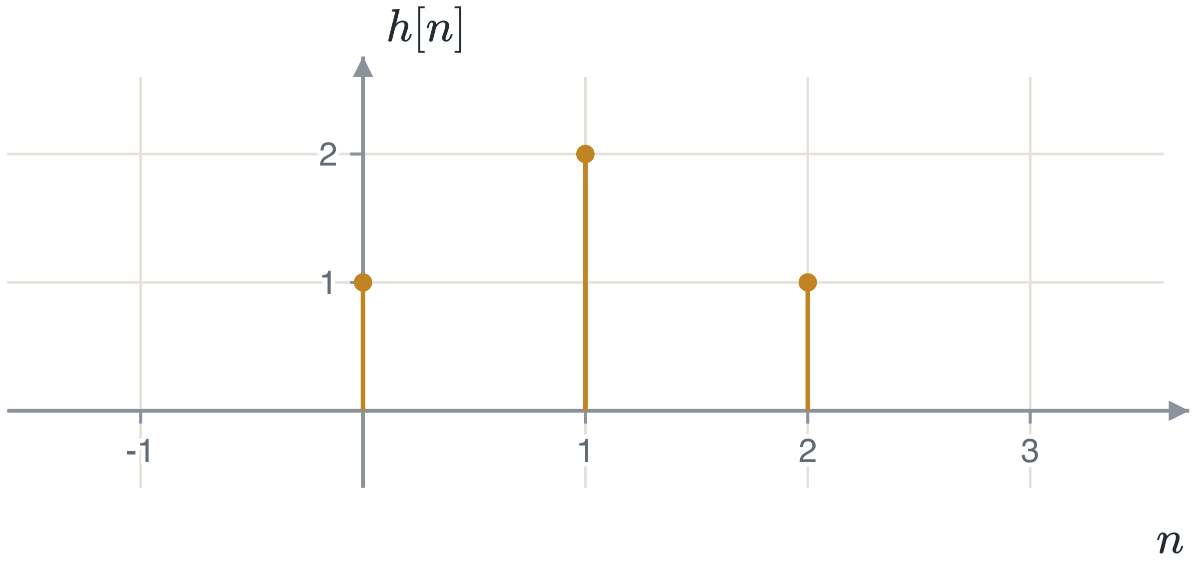
a) Compute and plot $y[n] = x[n] * h[n]$.

b) Verify that $y[n]$ is symmetric about $n = 0.5$, and say why that point is expected.

D3-07

The sequences $x[n]$ and $h[n]$ of a discrete-time LTI system are shown below.



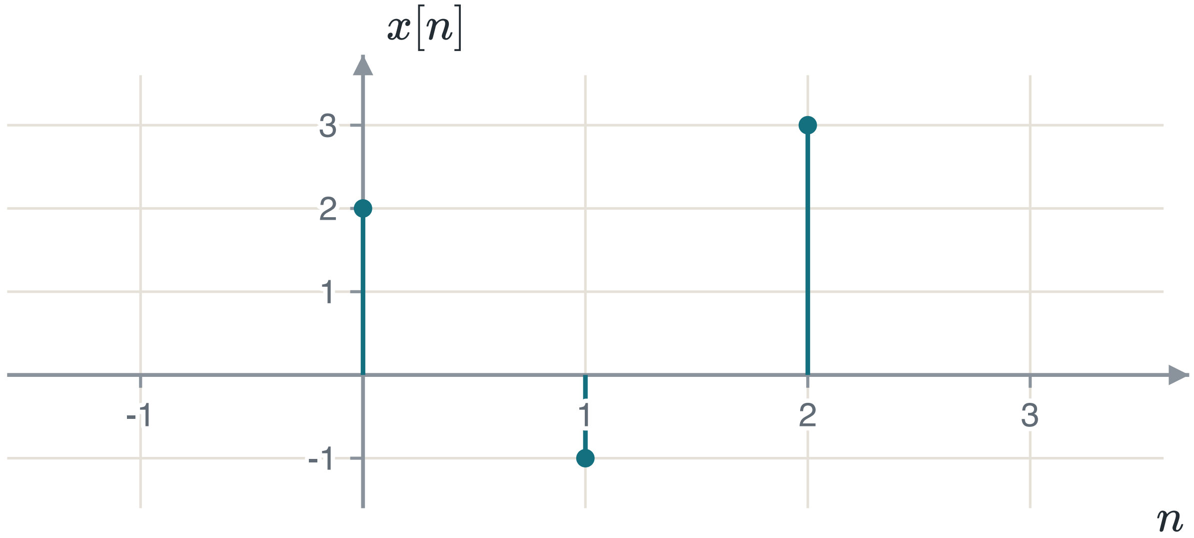


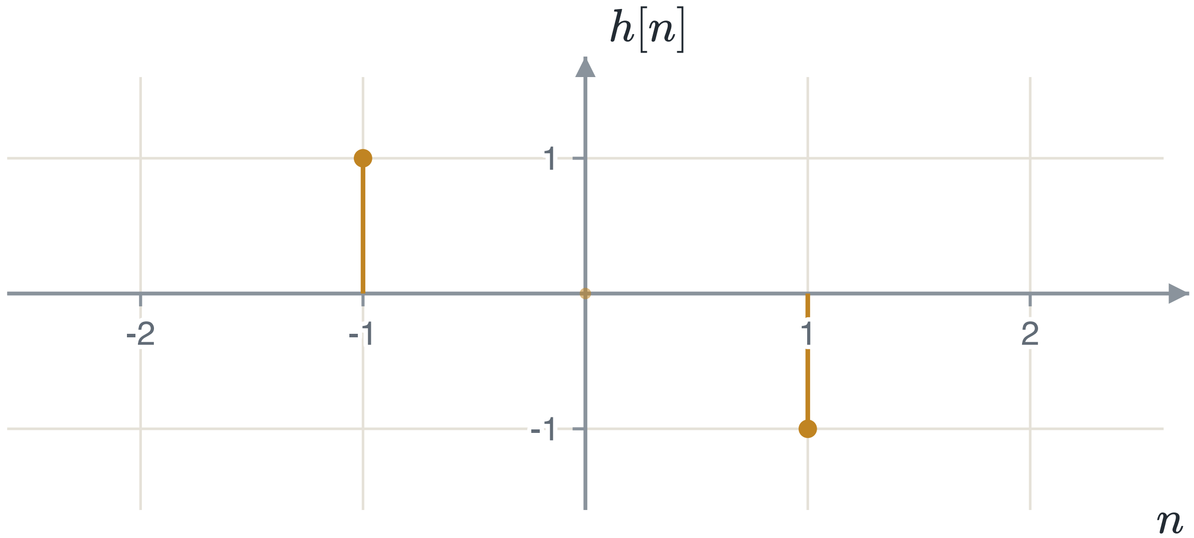
a) Compute and plot $y[n] = x[n] * h[n]$.

b) Confirm that the peak value of $y[n]$ equals $\sum_n h[n]$, and state the two indices where the peak occurs.

D3-08

The sequences $x[n]$ and $h[n]$ of a discrete-time LTI system are shown below.



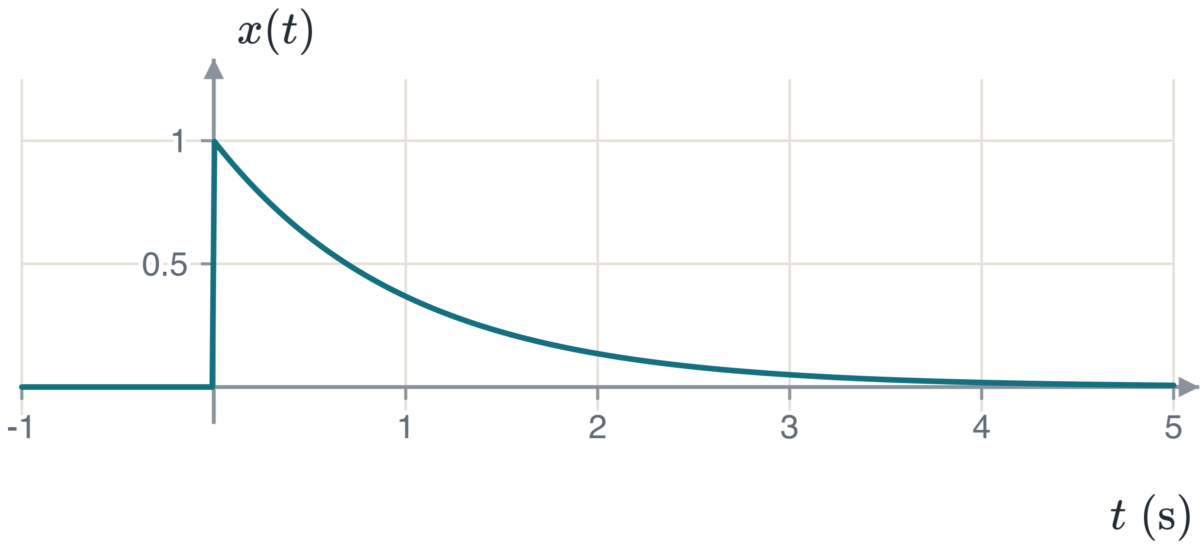


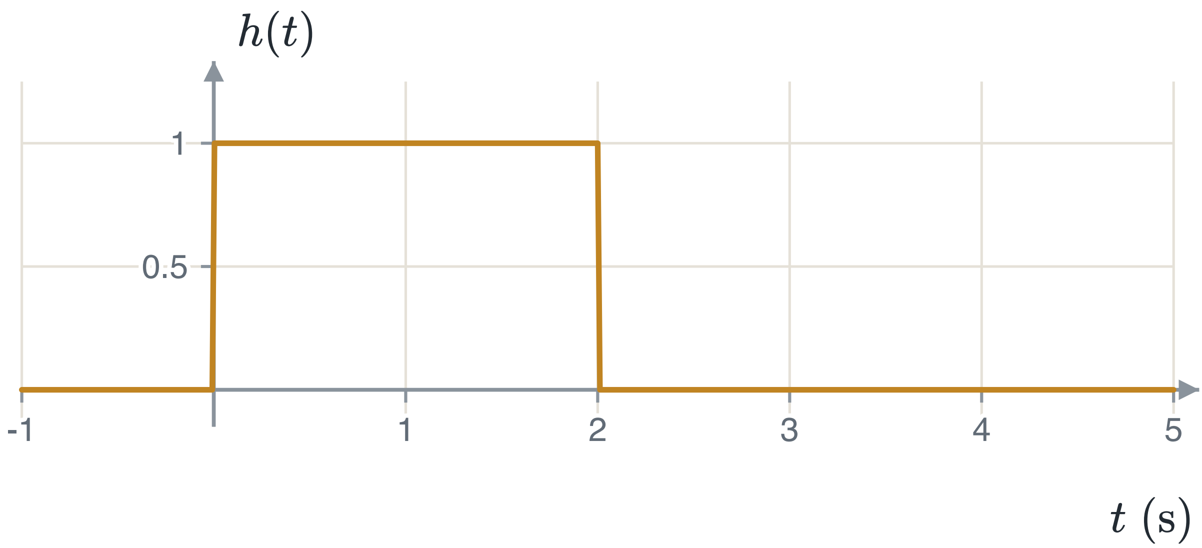
a) Compute and plot $y[n] = x[n] * h[n]$.

b) Show that $y[n] = x[n+1] - x[n-1]$ directly from the two non-zero locations of $h[n]$, and use it to check one sample of part (a) without repeating the full sum.

D3-09

Let $x(t) = e^{-t}u(t)$ and $h(t) = u(t) - u(t-2)$.





a) Compute $y(t) = x(t) * h(t)$ for $0 \leq t < 2$.

b) Compute $y(t)$ for $t \geq 2$.

c) Confirm the two branches agree at $t = 2$, and state $\lim_{t \to \infty} y(t)$.

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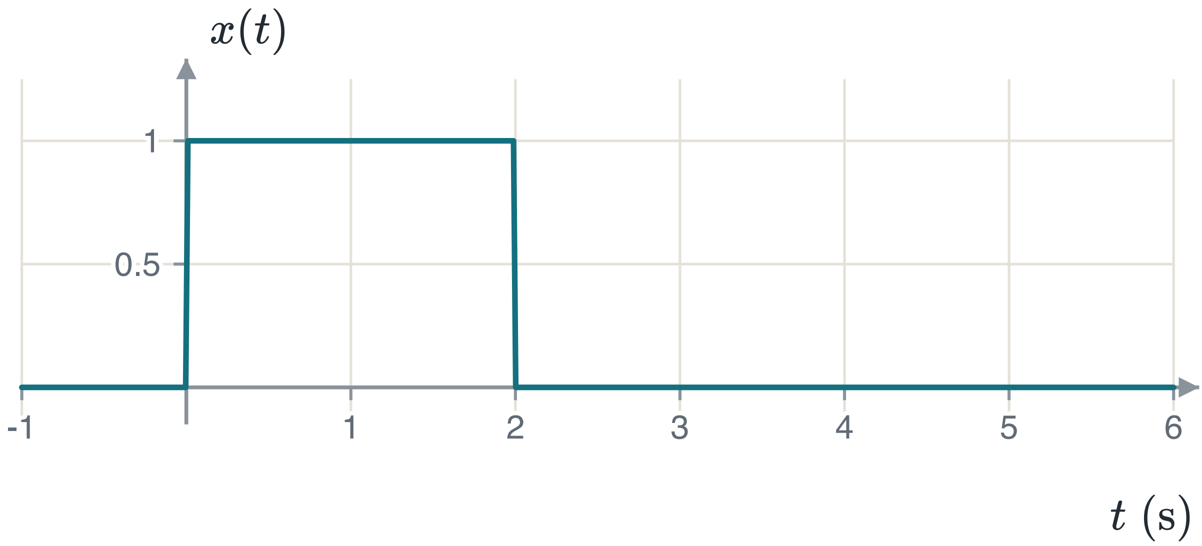
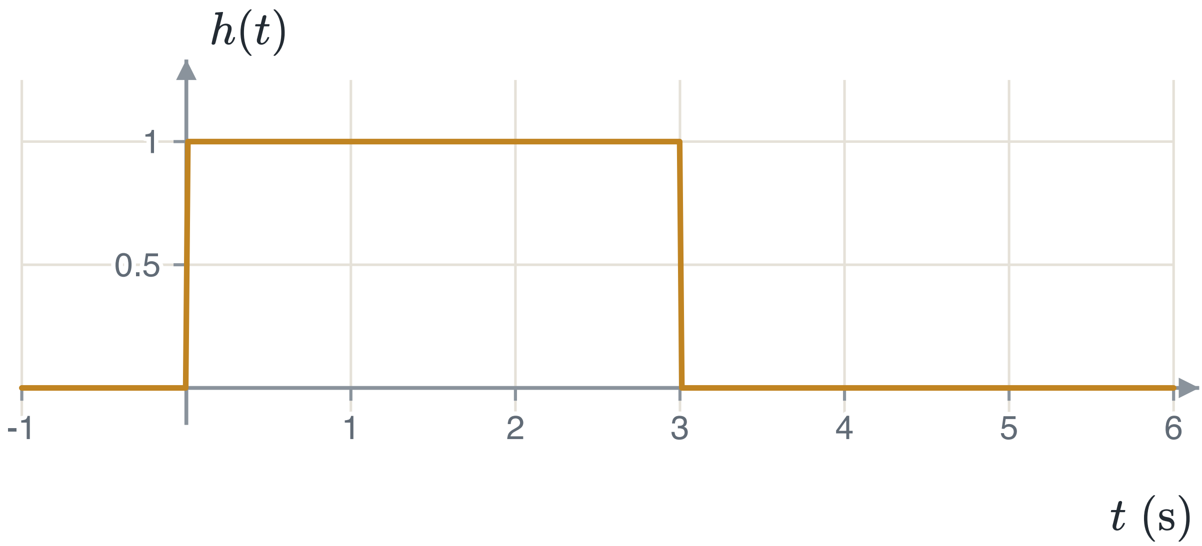
D3-10

Let $x(t) = 1$ for $0 < t < 2$ and $h(t) = 1$ for $0 < t < 3$, both zero elsewhere.





a) List the four case boundaries and say where each comes from.

b) Compute $y(t) = x(t) * h(t)$ in each of the three non-zero cases.

c) Verify that the total area under $y(t)$ equals the product of the two pulse widths.

D3-11

Let $x(t) = e^{-2t}u(t)$ and $h(t) = e^{-3t}u(t)$.

a) Compute $y(t) = x(t) * h(t)$ for $t \geq 0$, stating the case for $t < 0$ separately.

b) Find the time t^* at which $y(t)$ is maximum, and the peak value.

c) Evaluate $y(0)$ directly from the convolution integral, without using the closed form of part (a).

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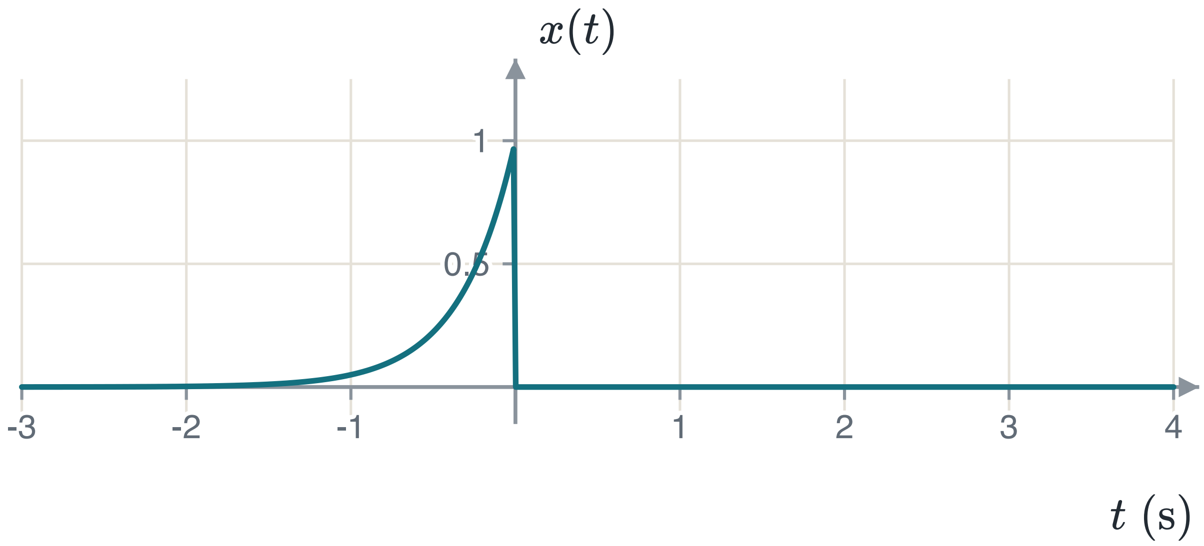
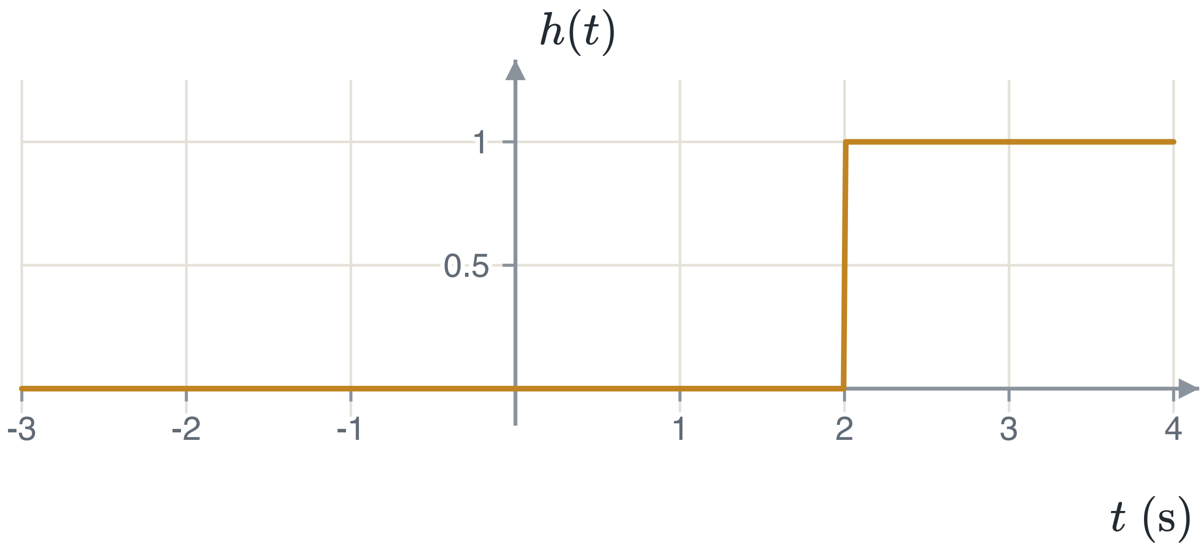
D3-12

Let $x(t) = e^{3t}u(-t)$ and $h(t) = u(t-2)$.





a) Compute $y(t) = x(t) * h(t)$ for $t < 2$.

b) Compute $y(t)$ for $t \geq 2$.

c) Confirm the two branches agree at $t = 2$, and interpret the system as a delayed accumulator.

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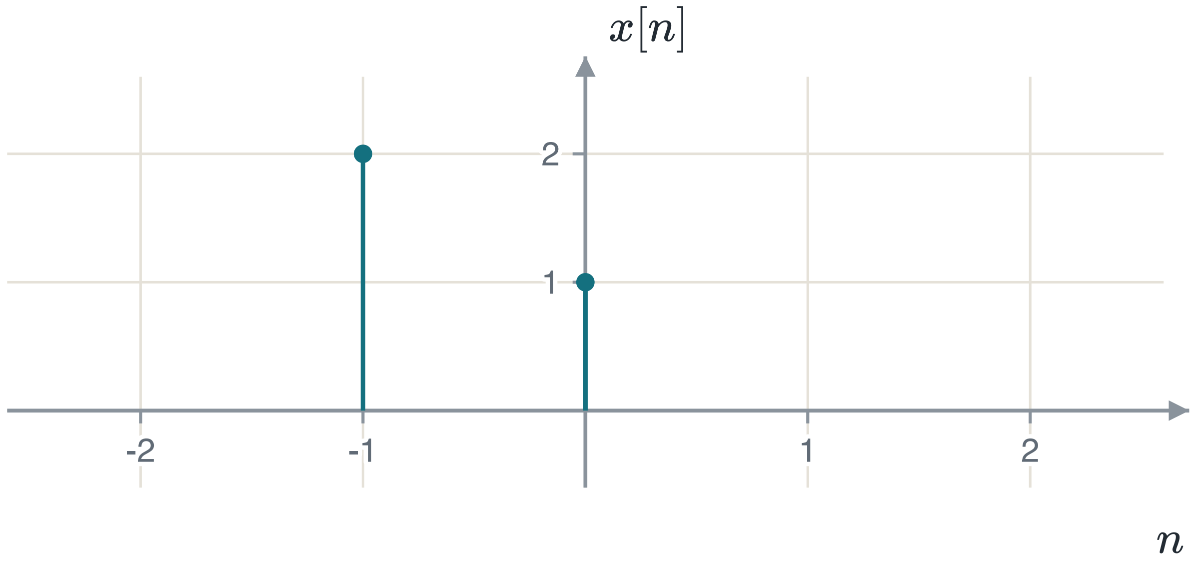
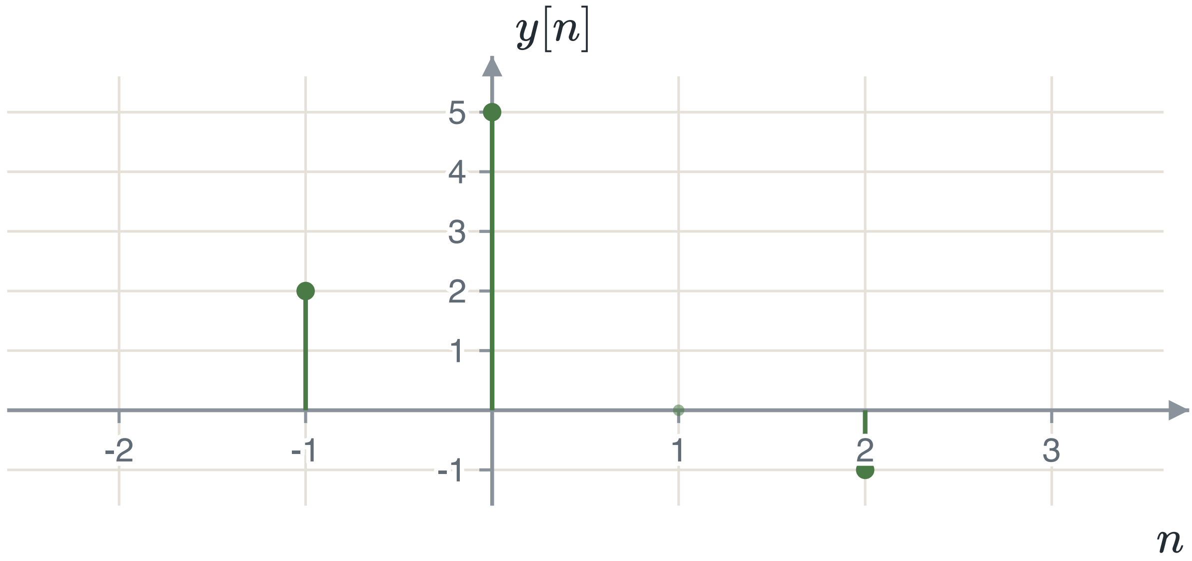
D3-13

A discrete-time LTI system has $x[n] = 2\delta[n+1] + \delta[n]$ at its input. The measured output $y[n]$ is shown below. The system is known to be causal, with an impulse response at most three samples long.





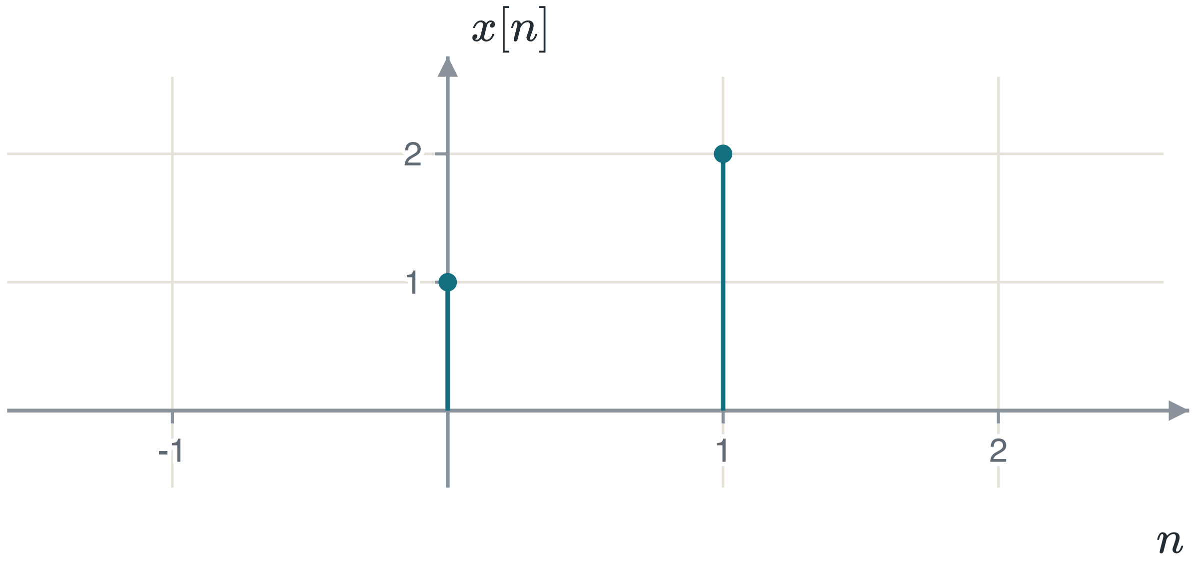
a) Using $y[-1]$ alone, find $h[0]$.

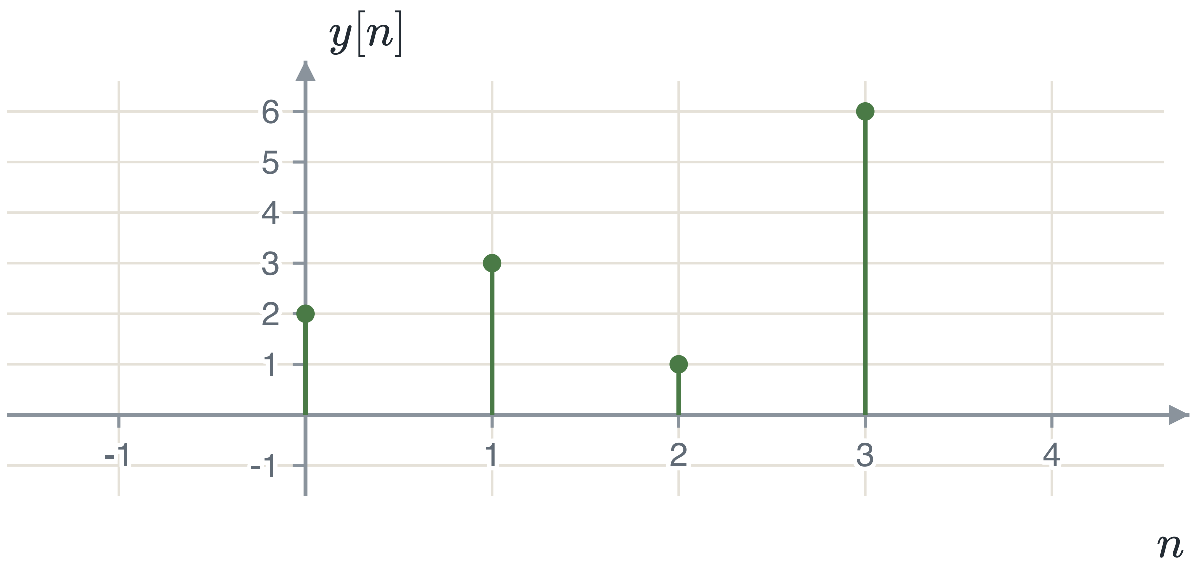
b) Using $y[0]$ and $h[0]$, find $h[1]$; then using $y[1]$, find $h[2]$.

c) Confirm $h[2]$ against $y[2]$, without introducing a new unknown.

D3-14

A discrete-time LTI system has $x[n] = \delta[n] + 2\delta[n-1]$ at its input. The measured output $y[n]$ is shown below. The system is known to be causal, with an impulse response at most three samples long.





a) Using $y[0]$ alone, find $h[0]$.

b) Using $y[1]$ and $h[0]$, find $h[1]$; then using $y[2]$, find $h[2]$.

c) Confirm $h[2]$ against $y[3]$, without introducing a new unknown.

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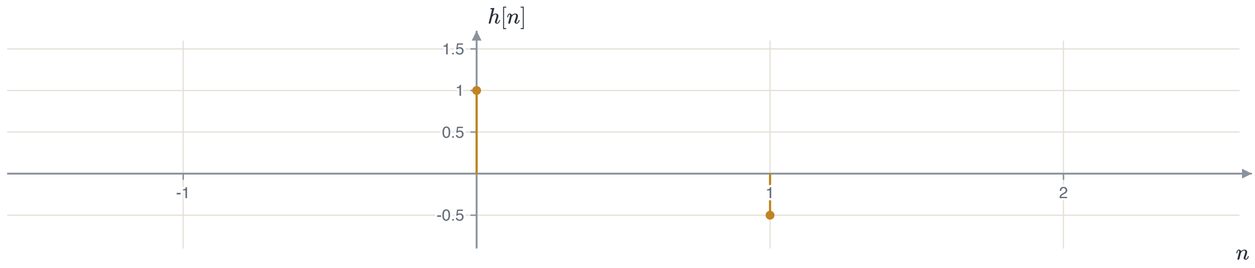
D3-15

A discrete-time LTI system has impulse response $h[n] = \delta[n] - \frac{1}{2}\delta[n-1]$, shown below.



a) Find the impulse response $g[n]$ of the causal inverse system, defined by $h[n] * g[n] = \delta[n]$.

b) Determine whether $g[n]$ is causal and whether it is stable.

c) Verify $h[n] * g[n] = \delta[n]$ directly at $n = 0, 1, 2$.

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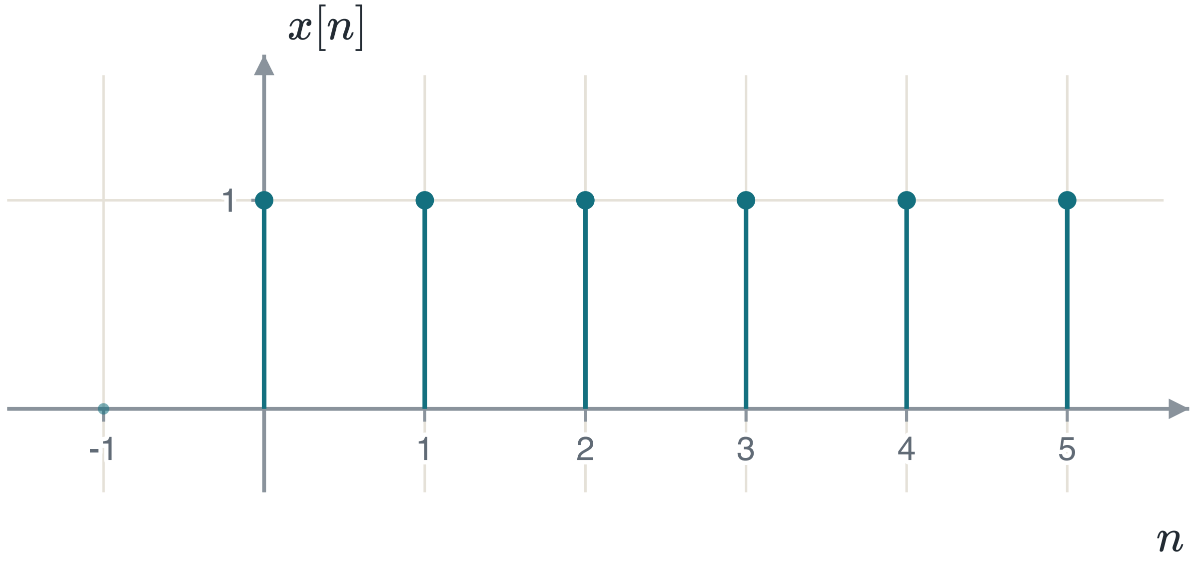
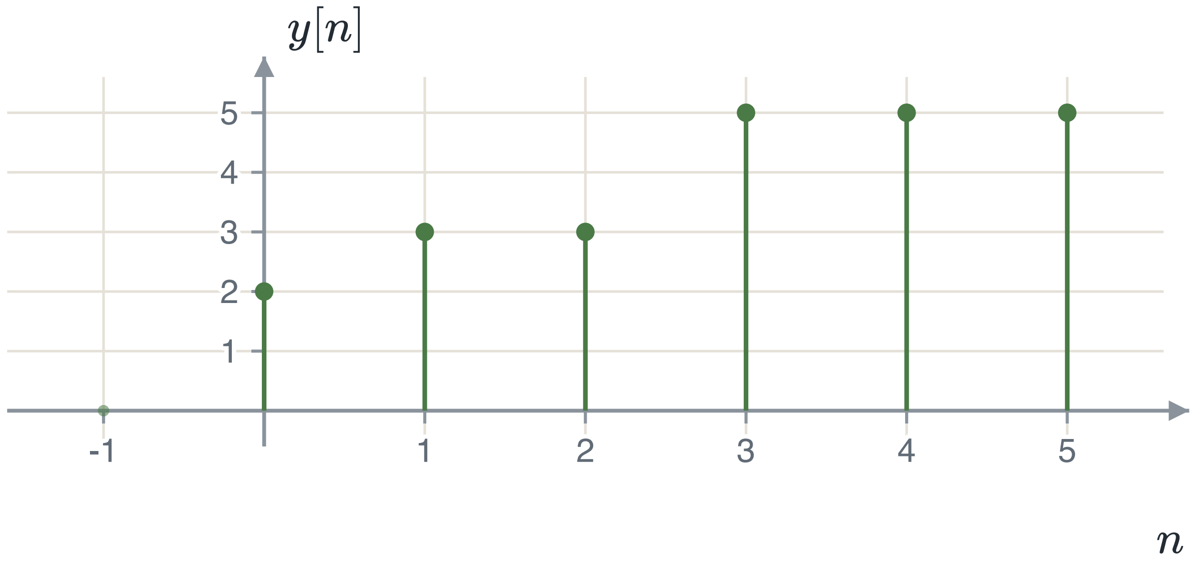
D3-16

A discrete-time LTI system is driven by the unit step $x[n] = u[n]$. The measured output $y[n]$ is shown below.





a) Explain why $h[n] = y[n] - y[n-1]$ when the input is $x[n] = u[n]$.

b) Use that relation to find $h[0]$, $h[1]$, $h[2]$, $h[3]$.

c) State $\sum_n h[n]$ directly from the plotted $y[n]$, without adding the four values of part (b).

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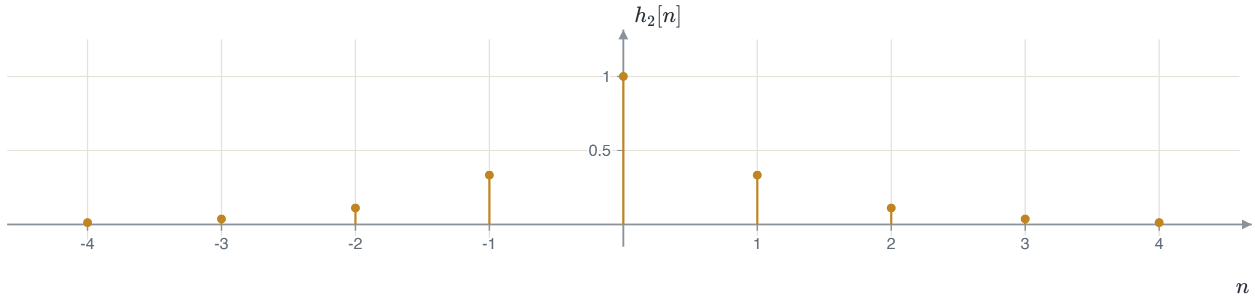
D3-17

Two discrete-time LTI systems have impulse responses

$$h_1[n] = 2^n u[n] \qquad \text{and} \qquad h_2[n] = \left(\tfrac{1}{3}\right)^{|n|}.$$

$h_2[n]$ is shown below; it is two-sided.



a) Determine whether $h_1[n]$ is causal, and whether it is stable.

b) Determine whether $h_2[n]$ is causal, and whether it is stable.

c) Say what the pairing of these two results shows about the two properties.

D3-18

A continuous-time LTI system has impulse response $h(t) = e^{2t}u(-t)$.

a) Determine whether the system is causal.

b) Determine whether it is stable.

c) For the bounded input $x(t) = u(t)$, compute $y(t) = x(t) * h(t)$ explicitly and confirm it stays within the bound predicted by stability.

D3-19

A discrete-time LTI system has impulse response $h[n] = (0.6)^n u[n]$.

a) Find and plot the step response $s[n] = h[n] * u[n]$.

b) Recover $h[n]$ from $s[n]$ using $h[n] = s[n] - s[n-1]$, and confirm it reproduces the given $h[n]$.

c) State $\lim_{n\to\infty} s[n]$ and identify it as $\sum_n h[n]$.

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D3-20

A continuous-time LTI system has impulse response $h(t) = e^{-4t}u(t)$.

a) Find and plot the step response $s(t) = h(t) * u(t)$.

b) Recover $h(t)$ from $s(t)$ using $h(t) = \mathrm{d}s/\mathrm{d}t$, and confirm it reproduces the given $h(t)$ for $t > 0$.

c) State $\lim_{t\to\infty} s(t)$ and identify it as $\int h(t)\,\mathrm{d}t$.

D3-21

The input and output relationship of an LTI system is described as

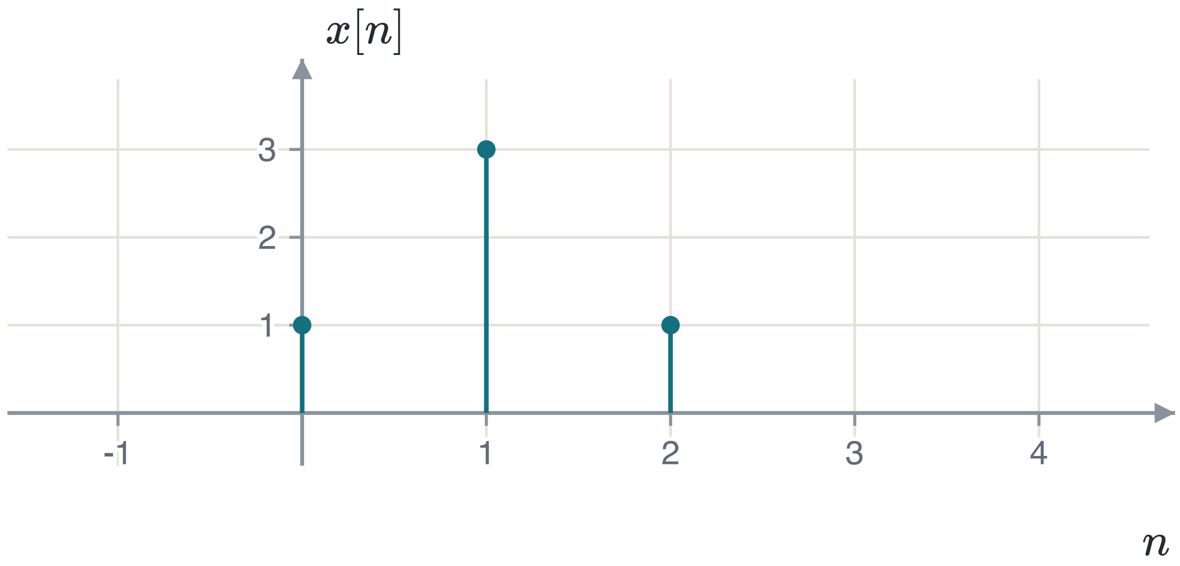
$$y[n] = 3x[n] + x[n-2] + 2x[n-3].$$

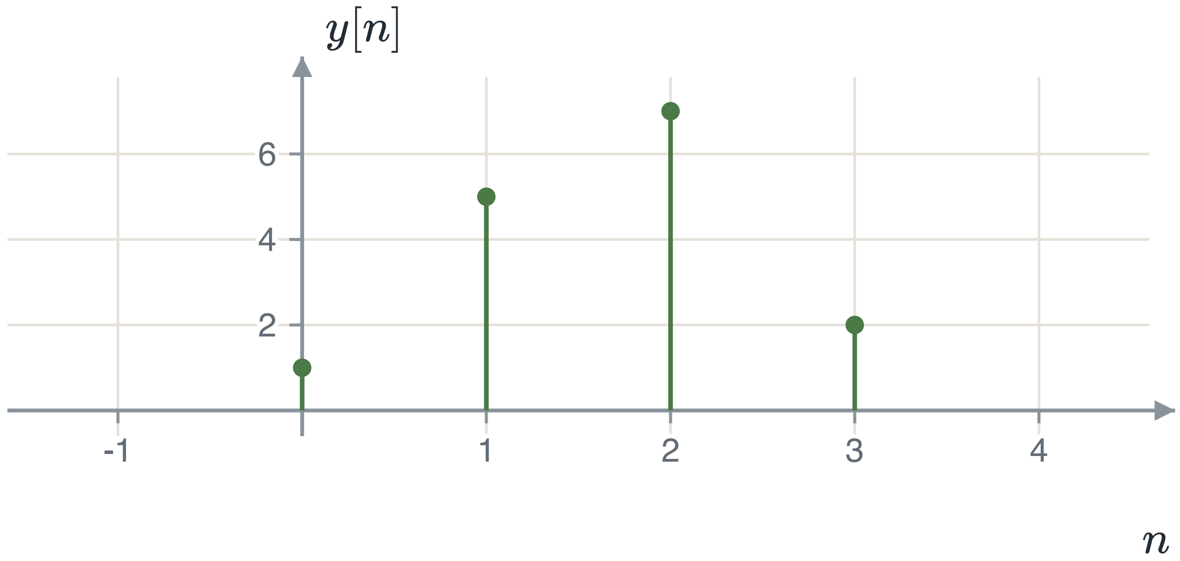
a) Determine and plot the impulse response of this LTI system, $h[n]$.

b) For the input signal $x[n] = \sum_{k=-1}^{2} k\,\delta[n-2k]$, compute and plot the output signal $y[n] = x[n] * h[n]$, using the discrete-time convolution.

D3-22

In the figure below, the signal $x[n]$ is input into a linear time-invariant (LTI) system, resulting in $y[n]$ as the output.





a) Determine and plot the impulse response of this LTI system, $h[n]$. *Hint: attempt to express $y[n]$ as a function of $x[n]$.*

b) Given the input signal $x_1[n] = 2\delta[n+1] - \delta[n] + \delta[n-2]$, sketch the output signal $y_1[n] = x_1[n] * h[n]$, using the discrete-time convolution.

D3-23

The input and output relationship of an LTI system is described as

$$y[n] = \tfrac{1}{3}y[n-1] + x[n].$$

a) Determine and plot the impulse response of this LTI system, $h[n]$.

b) For the input signal $x[n] = u[n]$, compute and plot the output signal $y[n] = x[n] * h[n]$, using the discrete-time convolution.

D3-24

Let $x(t) = e^{-2|t|}[u(t+1) - u(t-1)]$ be an input to the LTI system which has the following impulse response, $h(t) = u(t)$. Compute and plot the output signal $y(t) = x(t) * h(t)$, using the continuous-time convolution.

a) Write the convolution integral and identify the ranges of t that need separate treatment.

b) Evaluate the integral on each range and plot $y(t)$.

D3-25

Let

$$x(t) = \begin{cases} e^{-3t}, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

be an input to the LTI system which has the following impulse response, $h(t) = u(t) - u(t-2)$. Compute the output signal $y(t) = x(t) * h(t)$, using the continuous-time convolution.

a) Identify the ranges of t over which the overlap of $x(\tau)$ and $h(t-\tau)$ changes.

b) Evaluate the convolution integral on each range and plot $y(t)$.

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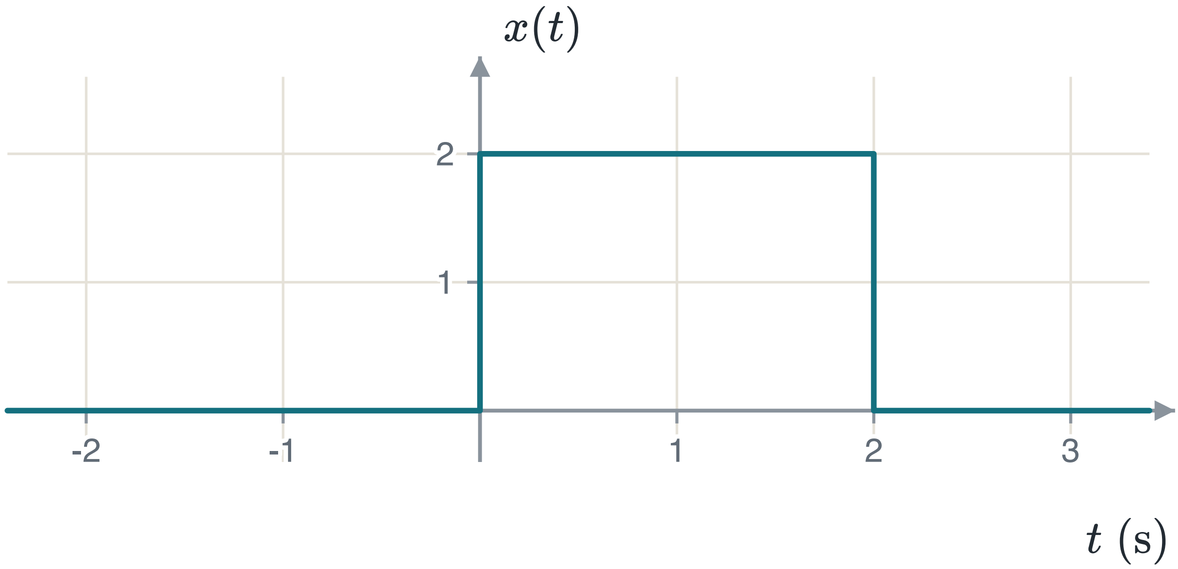
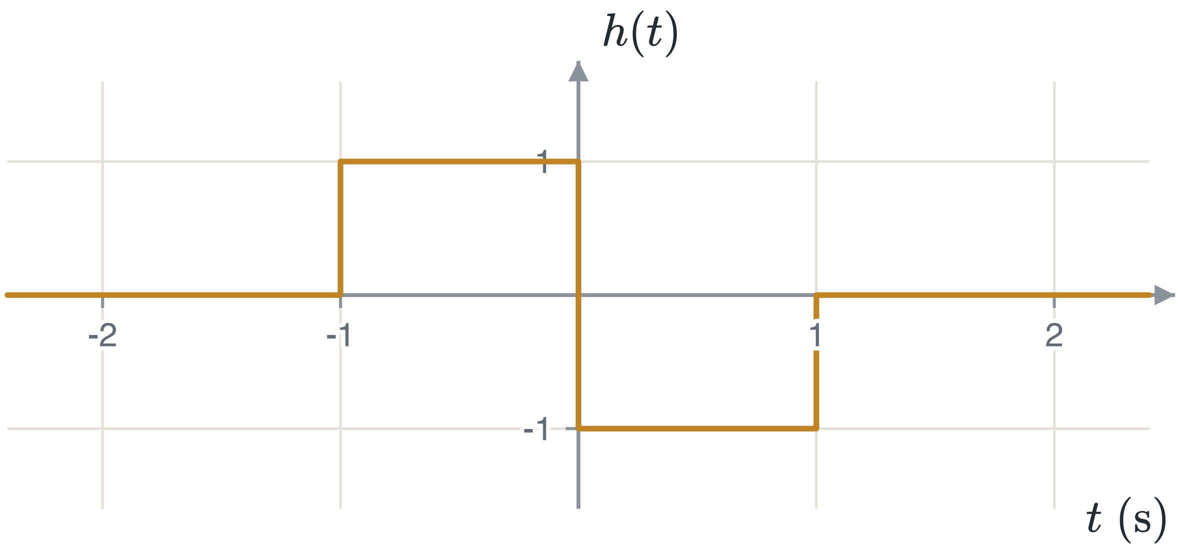
D3-26

The $x(t)$ signal sketched below is an input to the LTI system with the impulse response $h(t)$, also sketched below. Compute and plot the output signal $y(t) = x(t) * h(t)$, using the continuous-time convolution.



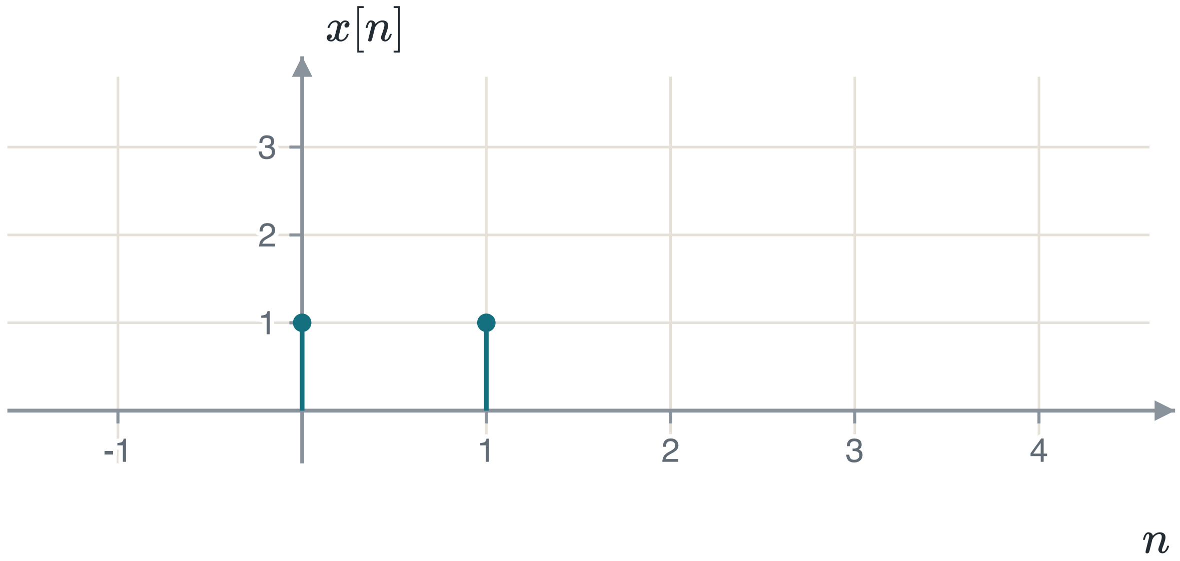


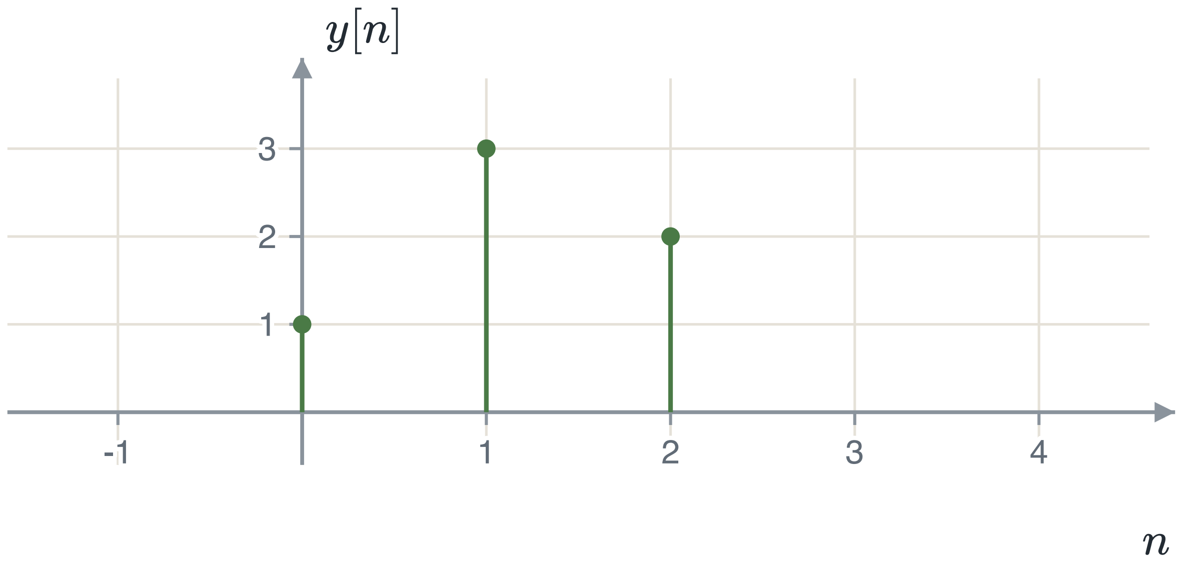
a) Write $h(t)$ in terms of step functions and give its running integral.

b) Evaluate the convolution and plot $y(t)$.

D3-27

Consider a discrete-time *causal* LTI system where the output signal $y[n]$ is obtained when the input signal $x[n]$ is applied to this system. Both are plotted below.





a) Calculate and plot the impulse response of this LTI system.

b) Determine and plot the output signal if the input signal is defined as $x_2[n] = (-1)^n$ for $0 \leq n \leq 3$, and zero otherwise.

D3-28

Consider two discrete-time systems with input $x[n]$ and output $y[n]$, each defined by a difference equation and each initially at rest:

$$\mathrm{S}_1: \; y[n] = x[n] + 2y[n-1], \qquad \mathrm{S}_2: \; y[n] = x[n] + \tfrac{1}{2}y[n-1].$$

a) Plot the impulse response of S_1.

b) For the impulse response obtained in part (a), state whether the system is stable or not.

c) Repeat both parts for S_2, and say what decides the difference.

D3-29

An LTI system has the impulse response $h[n] = \delta[n] - \delta[n-2]$.

a) Write the input-output difference equation of the system.

b) For the input $x[n] = u[n] - u[n-4]$, compute and plot the output $y[n] = x[n] * h[n]$.

c) State whether the system is causal and whether it is stable.

D3-30

Let $x(t) = u(t) - u(t-2)$ be an input to the LTI system which has the impulse response $h(t) = e^{-t}u(t)$.

a) Identify the ranges of t over which the overlap changes, and write the convolution integral on each.

b) Evaluate the integral and plot $y(t) = x(t) * h(t)$.

c) State the value $y(t)$ approaches as $t \to \infty$ and explain it.

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MODULE

4

Fourier Series

30 questions on fourier series. Write your reasoning in the space under each one.

D4-01

Let

$$x(t) = 4 + 2\cos(2\pi t) + 6\sin\left(\tfrac{2\pi}{3}t\right).$$

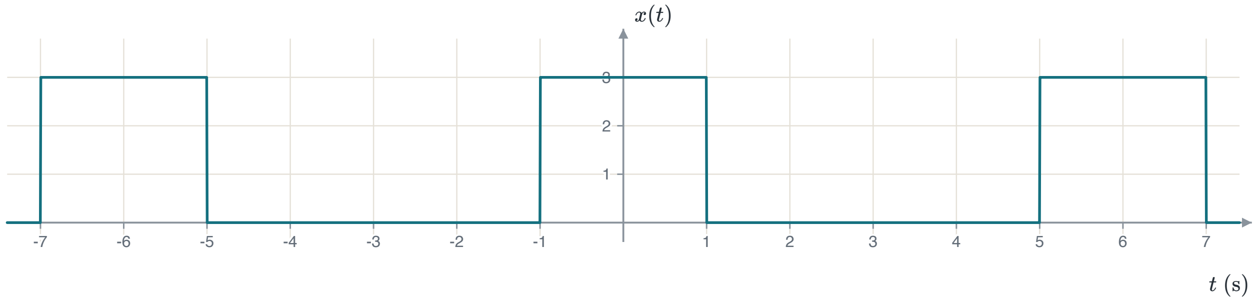
a) Find the fundamental period T_0 and ω_0, and state which harmonic each term is.

b) Find a_k for every k, and plot the magnitude spectrum.

c) Plot the phase spectrum.

D4-02

A periodic signal has $T_0 = 6$ and equals 3 for $|t| < 1$ and 0 for $1 < |t| < 3$ inside one period.



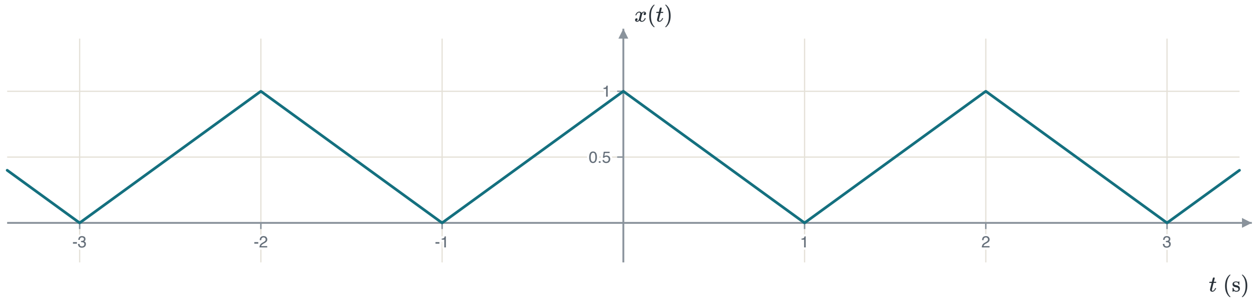
a) Find a_0 by inspection.

b) Find a_k for $k \neq 0$ using the analysis equation.

c) Plot $|a_k|$ for $-6 \leq k \leq 6$ and say which harmonics are absent.

D4-03

A periodic signal has $T_0 = 2$ and equals $x(t) = 1 - |t|$ for $-1 \le t \le 1$ inside one period: a triangular pulse of height 1, with no jump anywhere, only a corner at $t = 0$ and at $t = \pm 1$.



a) Find a_0 without integrating.

b) Find a_k for $k \neq 0$ using integration by parts.

c) Plot $|a_k|$ for $-7 \le k \le 7$ and state how fast the envelope decays.

D4-04

A real periodic signal $x(t)$ has fundamental angular frequency $\omega_0 = 2$ rad/s. Its only non-zero Fourier series coefficients are

$$a_0 = -1, \qquad a_2 = 3j, \quad a_{-2} = -3j, \qquad a_5 = 2e^{j\pi/4}, \quad a_{-5} = 2e^{-j\pi/4}.$$

a) Using the conjugate-pair reassembly, write $x(t)$ as a sum of real cosines.

b) Evaluate $x(0)$ from this real form.

D4-05

Let

$$x(t) = -3 + 4\cos(5t) - 2\sin(15t).$$

a) Find the Fourier series coefficients a_k.

b) Find the average power over one period using Parseval's relation.

c) Confirm the same value using the direct amplitude rule for a sum of sinusoids.

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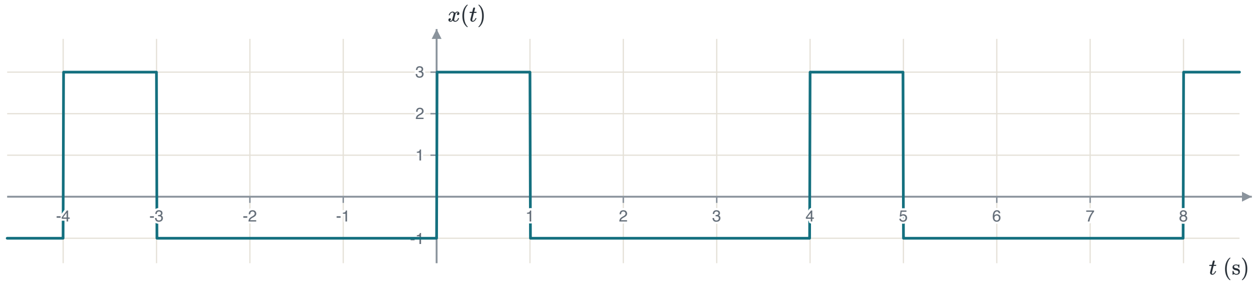
D4-06

A periodic signal has $T_0 = 4$ and equals $x(t) = 3$ for $0 < t < 1$ and $x(t) = -1$ for $1 < t < 4$ inside one period.



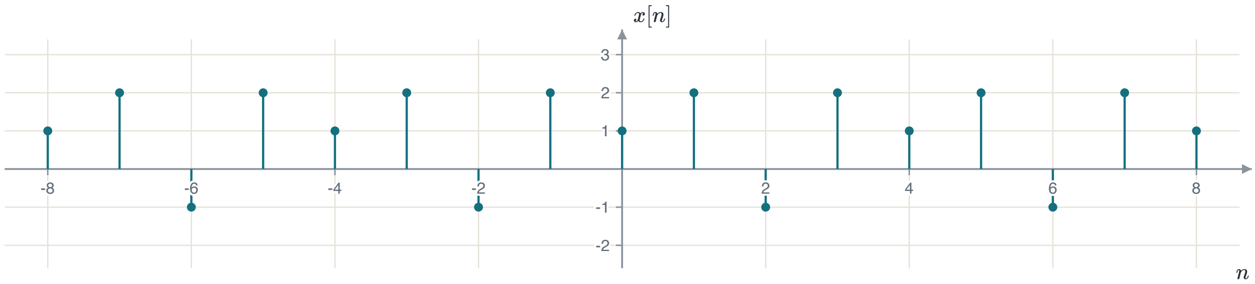
a) Find the average power directly, using the period $0 \le t < 4$.

b) Recompute the average power using the period $-2 \le t < 2$, and confirm the two agree.

D4-07

A periodic sequence has $N = 4$ and one period given by

$$x[0] = 1, \quad x[1] = 2, \quad x[2] = -1, \quad x[3] = 2.$$



a) Find a_k for $k = 0, 1, 2, 3$ using the analysis sum.

b) Find the average power directly in the time domain.

c) Confirm the same value using Parseval's relation.

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D4-08

A periodic signal has $T_0 = 2$ and equals $x(t) = 1 - |t|$ for $-1 \le t \le 1$ inside one period — the triangular pulse whose coefficients are $a_0 = \frac{1}{2}$ and $a_k = \dfrac{1-(-1)^k}{(k\pi)^2}$ for $k \neq 0$.

a) Find the average power P directly, by integrating x^2 over one period.

b) Find P again as an infinite sum over the coefficients, using $\displaystyle\sum_{k \text{ odd}, k\ge 1} \frac{1}{k^4} = \frac{\pi^4}{96}$.

c) Find the fraction of P already captured by the DC term and the first harmonic alone, and compare with a signal that has a jump.

D4-09

An LTI system has impulse response $h(t) = 5e^{-5t}u(t)$.

a) Find the frequency response $H(j\omega)$.

b) Find $|H(j\omega)|$ and $\angle H(j\omega)$ in closed form.

c) Plot the magnitude spectrum, and evaluate it at $\omega = 5$ rad/s.

D4-10

The periodic signal $x(t) = 2 + 3\cos(4t)$ is the input to an LTI system with impulse response $h(t) = 3e^{-3t}u(t)$.

a) Find the frequency response $H(j\omega)$.

b) Find the Fourier series coefficients of $x(t)$.

c) Find the Fourier series coefficients of the output $y(t)$.

d) Write $y(t)$ in real form.

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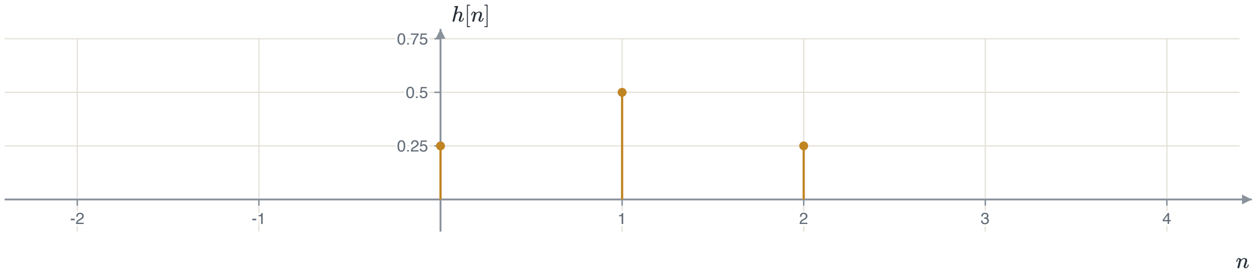
D4-11

A discrete-time LTI system has impulse response

$$h[n] = 0.25\,\delta[n] + 0.5\,\delta[n-1] + 0.25\,\delta[n-2].$$



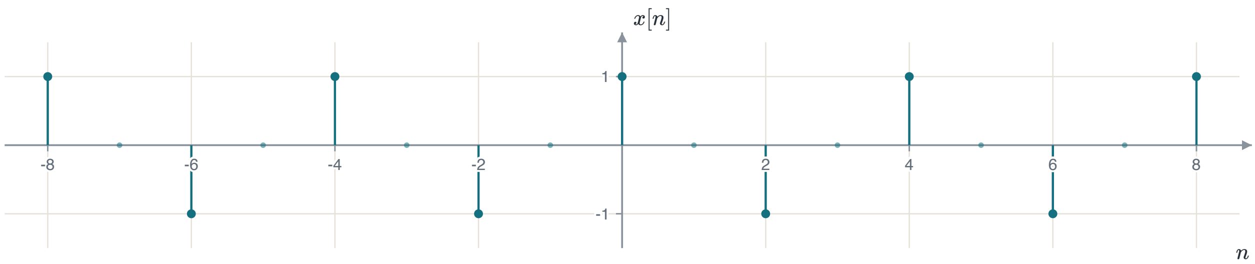
a) Find $H(e^{j\omega})$.

b) Show that $|H(e^{j\omega})| = \cos^2(\omega/2)$, and plot the magnitude spectrum over one period.

c) Evaluate H at $\omega = 0$ and at $\omega = \pi$, and say what kind of filter this is.

D4-12

The periodic sequence $x[n] = \cos\left(\frac{\pi}{2}n\right)$ is the input to a discrete-time LTI system with impulse response $h[n] = 0.6\,\delta[n] + 0.4\,\delta[n-1]$.



a) Find the fundamental period N and the coefficients a_k of $x[n]$.

b) Find $H(e^{j\omega})$ and evaluate it at $\omega = \omega_0$.

c) Find the output coefficients and write $y[n]$ in real form.

d) Plot $y[n]$ as a stem for $-8 \le n \le 8$.

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D4-13

Let

$$x[n] = 3\cos\left(\tfrac{\pi}{4}n\right) - \sin\left(\tfrac{2\pi}{3}n\right).$$

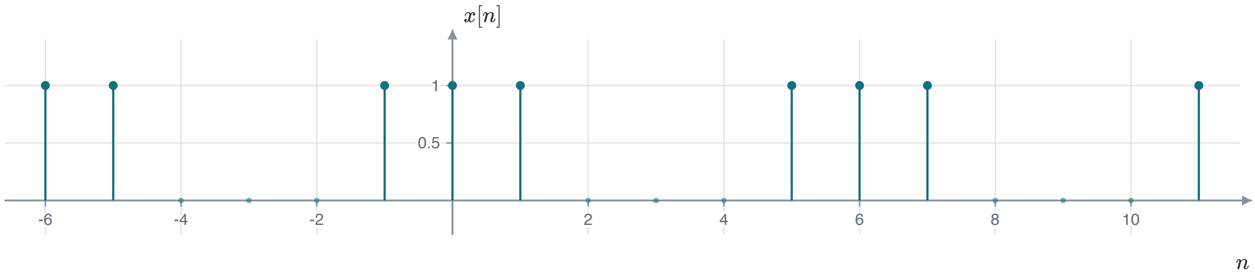
a) Find the fundamental period N_0 and ω_0, and state which harmonic each term is.

b) Find every non-zero coefficient.

c) Plot $|a_k|$ over one period, $-12 \le k \le 11$.

D4-14

A periodic sequence has $N = 6$ and equals $1, 1, 0, 0, 0, 1$ for $n = 0, 1, 2, 3, 4, 5$ inside one period.



a) List $x[n]$ for one full period and identify which samples are non-zero.

b) Find a_k for $k = 0, 1, 2, 3$ using the analysis sum.

c) Plot $|a_k|$ over one period and confirm a_0 equals the mean of the sequence.

D4-15

A real periodic sequence $x[n]$ has fundamental period $N = 5$. Its only non-zero coefficients are

$$a_0 = 2, \qquad a_1 = 1 + j, \qquad a_4 = 1 - j.$$

a) Using the conjugate-pair reassembly, write $x[n]$ as a single real cosine plus a constant.

b) Evaluate $x[0]$ directly from the three given coefficients, and from the real form.

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D4-16

The period-6 sequence $x[0] = 1, x[1] = 1, x[2] = 0, x[3] = 0, x[4] = 0, x[5] = 1$ has Fourier series coefficients $a_0 = \frac{1}{2}$, $a_1 = a_5 = \frac{1}{3}$, $a_2 = a_4 = 0$, $a_3 = -\frac{1}{6}$.

a) Reconstruct $x[0]$ using the synthesis sum over all six coefficients, and confirm it equals the given value.

b) Reconstruct $x[2]$ the same way, using all six coefficients.

c) Set a_3 to zero, keep the other five coefficients unchanged, and recompute $x[2]$. State the size of the resulting error.

D4-17

A real periodic signal $x(t)$ has fundamental angular frequency $\omega_0 = \pi$ rad/s and coefficients $a_1 = 2$, $a_2 = 1 - j$ (with $a_{-1} = 2$, $a_{-2} = 1 + j$ by conjugate symmetry). Let $y(t) = x(t - t_0)$ with $t_0 = 0.5$ s.

a) State the general time-shift property, with the correct sign in the exponent.

b) Compute b_1 and b_2, and confirm $|b_k| = |a_k|$ for both.

c) Compute $\angle b_2 - \angle a_2$ and confirm it equals $-2\omega_0 t_0$, modulo 2π.

D4-18

A periodic signal $x(t)$ with fundamental angular frequency $\omega_0 = 5$ rad/s is real and even: $x(t) = x(-t)$ for every t, and $x(t)$ is real-valued.

a) State what the time-reversal property alone gives for a_{-3}, in terms of a_3.

b) State what conjugation (realness) alone gives for a_{-3}, in terms of a_3.

c) Combine the two results to show that a_3 must be real, and decide whether $a_3 = 2 - 5j$ is a value a_3 could actually take.

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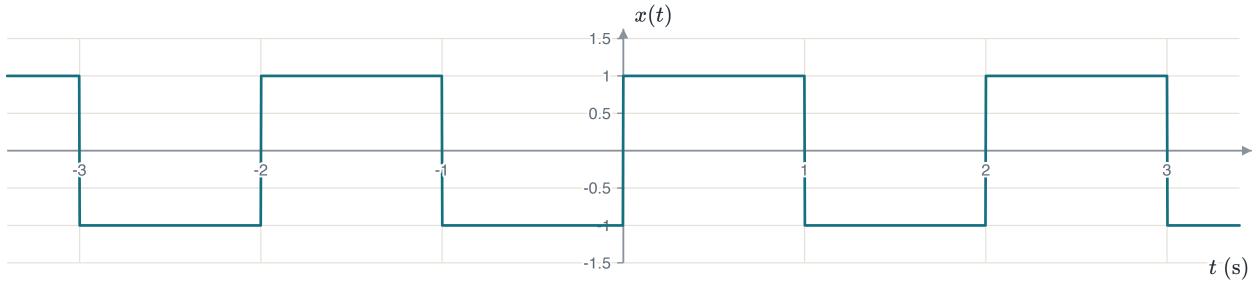
D4-19

A periodic signal has $T_0 = 2$ and equals $x(t) = -1$ for $-1 < t < 0$ and $x(t) = 1$ for $0 < t < 1$ inside one period: a real, odd, bipolar square wave.



a) State, without integrating, why $a_0 = 0$.

b) Derive a_k for $k \neq 0$ using the analysis equation, splitting the integral at $t = 0$.

c) Confirm every coefficient is purely imaginary, and plot the magnitude spectrum.

D4-20

A real periodic signal $x(t)$ has fundamental angular frequency $\omega_0 = 1$ rad/s. Its only non-zero coefficients are

$$a_0 = 1, \qquad a_3 = 2e^{j\pi/3}, \qquad a_5 = 1 - j.$$

a) Use conjugate symmetry to state a_{-3} and a_{-5}.

b) Write $x(t)$ in real closed form.

c) Compute the average power two independent ways, and confirm they agree.

D4-21

Let

$$x[n] = 2 + (-1)^n \qquad \text{and} \qquad y[n] = \sin\left(\tfrac{4\pi}{5}n + \tfrac{\pi}{2}\right)$$

be two periodic signals.

a) Determine the Fourier series coefficients and plot the magnitude of each coefficient for $x[n]$.

b) Determine the Fourier series coefficients and plot the magnitude of each coefficient for $y[n]$.

c) Let $z[n] = x[n]y[n]$ be a periodic signal with fundamental period 10. Determine the Fourier series coefficients and plot the magnitude of each coefficient for $z[n]$.

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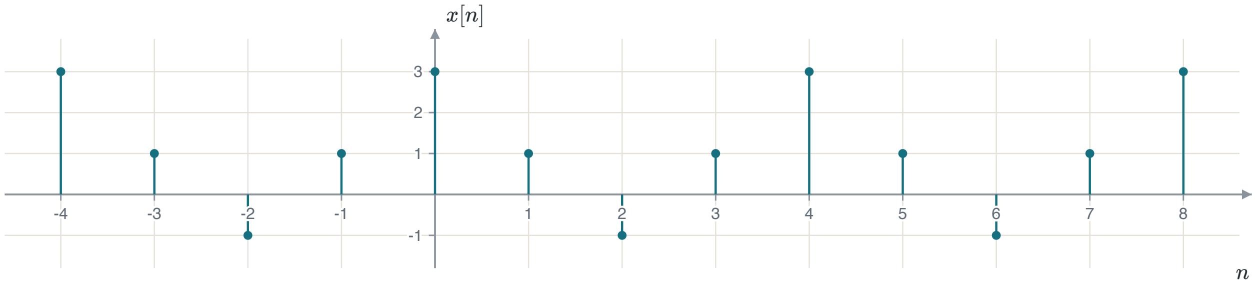
D4-22

Let $x[n]$ be a periodic signal with fundamental period $N = 4$, sketched below.



a) Plot the Fourier series coefficients of $x[n]$.

b) Compute the Fourier series coefficients and plot the magnitude of each coefficient for the signal $y[n] = x[n] - x[n-1]$.

c) Calculate the average power of the $y[n]$ signal defined in part (b).

D4-23

Let $x(t)$ be a periodic signal defined as

$$x(t) = 6\cos^2\left(\frac{\pi}{4}t\right) - 4\sin\left(\frac{\pi}{3}t\right).$$

a) Calculate the fundamental period of $x(t)$.

b) Plot the magnitude and phase of each set of Fourier series coefficients of $x(t)$.

c) Calculate the average power in one period of the periodic signal $x(t)$.

D4-24

Let $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$ be the input of a discrete-time LTI system with impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Let $y[n] = x[n] * h[n]$ be the output signal.

a) Determine the Fourier series coefficients and plot the magnitude of each coefficient for $x[n]$.

b) Determine the frequency response of $h[n]$ and plot its magnitude spectrum.

c) Determine the Fourier series coefficients and plot the magnitude of each coefficient for $y[n]$.

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D4-25

Let

$$x(t) = 1 + 2\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right) + 3\sin(\pi t)$$

be a periodic signal that is applied to a linear time-invariant (LTI) system with the impulse response $h(t) = \delta(t) - 2e^{-2t}u(t)$. At the output of the LTI system, the $y(t)$ signal is obtained.

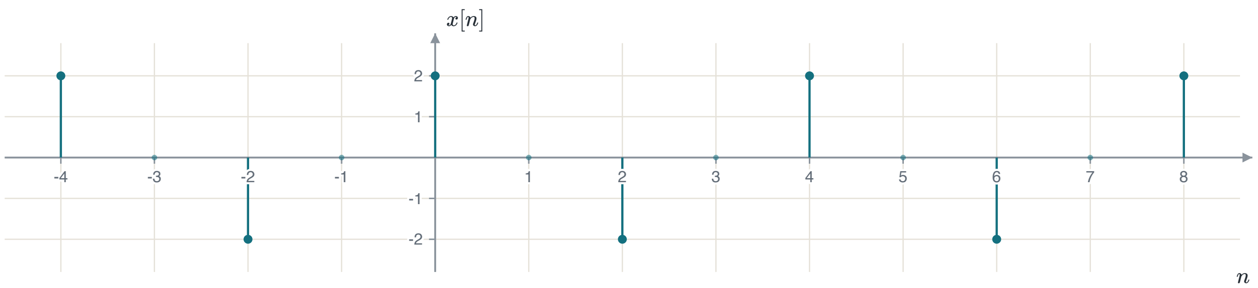
a) Determine the Fourier series coefficients and plot the magnitude of each coefficient for $x(t)$.

b) Determine the frequency response of $h(t)$.

c) Determine the Fourier series coefficients and plot the magnitude of each coefficient for $y(t)$.

D4-26

The periodic signal $x[n]$ sketched below is the input of a discrete-time LTI system with the impulse response $h[n] = u[n+1] - u[n-2]$. Let $y[n] = x[n] * h[n]$ be the output signal.



a) Plot the Fourier series coefficients of $x[n]$.

b) Determine the frequency response of $h[n]$.

c) Plot the Fourier series coefficients of $y[n]$.

D4-27

Let

$$a_k = \begin{cases} 2, & k \text{ even} \\ 1, & k \text{ odd} \\ 0, & |k| \geq 3 \end{cases}$$

be the Fourier series coefficients of a continuous-time signal $x(t)$ which is periodic with 6 seconds.

a) Find the Fourier series representation of $x(t)$, and write it as a sum of real cosines.

b) Calculate the average power of $x(t)$.

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D4-28

A periodic signal $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-6k]$ is the input of a discrete-time LTI system with the impulse response

$$h[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}.$$

Let $y[n] = x[n] * h[n]$ be the output signal.

a) Plot the Fourier series coefficients of $x[n]$.

b) Plot the frequency response of $h[n]$.

c) Calculate the average power in one period for the periodic signal $y[n]$.

D4-29

Let $x(t)$ be the periodic square wave of period $T_0 = 4$ seconds defined over one period by

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & 1 < |t| < 2. \end{cases}$$

a) Give the fundamental frequency ω_0 and the Fourier series coefficients a_k.

b) State which coefficients vanish and say why.

c) Calculate the average power of $x(t)$, and verify it against Parseval.

D4-30

Let

$$x(t) = 3 + 2\cos(2t) + \cos(4t)$$

be the input to an LTI system with impulse response $h(t) = e^{-t}u(t)$, and let $y(t)$ be the output.

a) Give the fundamental period of $x(t)$ and its Fourier series coefficients.

b) Determine the frequency response $H(j\omega)$ and evaluate it at each harmonic present.

c) Determine the Fourier series coefficients of $y(t)$ and plot their magnitudes.

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MODULE

5

Continuous-Time Fourier Transform

30 questions on continuous-time fourier transform. Write your reasoning in the space under each one.

D5-01

Find the Fourier transform of

$$x(t) = e^{-4t}u(t).$$

a) Compute $X(j\omega)$, stating the condition under which the integral converges.

b) Evaluate $|X(j3)|$ and $\angle X(j3)$, and sketch $|X(j\omega)|$.

c) Check $X(0)$ against the area of $x(t)$.

D5-02

Find the Fourier transform of

$$x(t) = t\,e^{-3t}u(t).$$

a) Compute $X(j\omega)$ directly from the analysis equation.

b) Give $|X(j\omega)|$, state where it peaks, and sketch it.

c) Check $X(0)$ against the area of $x(t)$.

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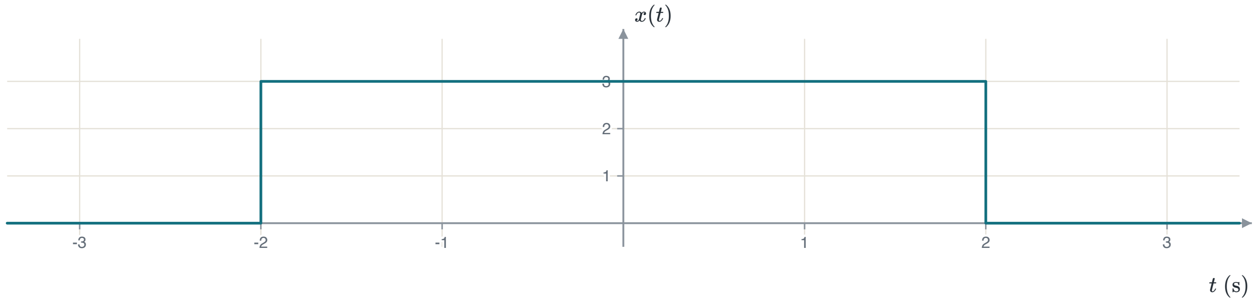
D5-03

A rectangular pulse of height 3 is defined by

$$x(t) = \begin{cases} 3, & |t| < 2, \\ 0, & |t| > 2. \end{cases}$$



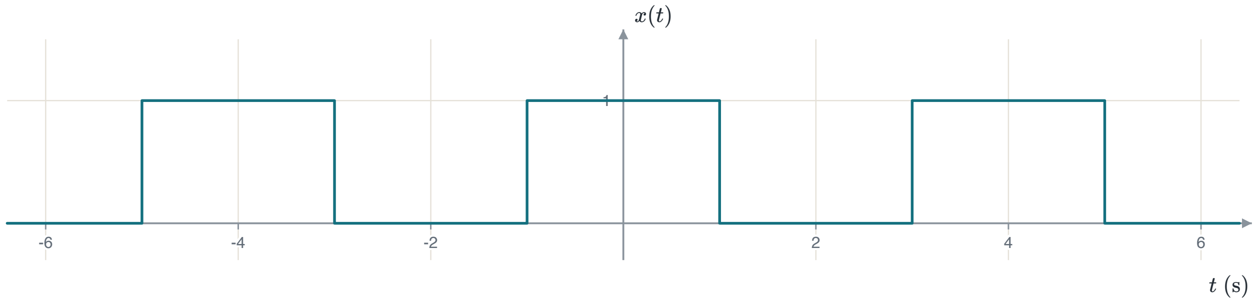
a) Compute $X(j\omega)$ and write it using the unnormalised sinc.

b) Give the locations of the zeros of $X(j\omega)$.

c) Sketch $X(j\omega)$ and check $X(0)$ against the area of $x(t)$.

D5-04

A periodic rectangular pulse train has period $T_0 = 4$ and equals 1 on $|t| < 1$ inside each period, 0 elsewhere in the period.



a) Find the Fourier series coefficients a_k of $x(t)$, using the analysis equation over one period.

b) Give $X(j\omega)$ as a sum of impulses, and state the weight at $k = 0, \pm1, \pm2, \pm3$.

c) Sketch $X(j\omega)$ for $-2\pi \le \omega \le 2\pi$, showing the negative frequencies, and say which of the four impulses vanish.

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D5-05

Find $x(t)$ given

$$X(j\omega) = \frac{1}{(j\omega + 2)(j\omega + 5)}.$$

a) Split X into partial fractions.

b) Give $x(t)$.

c) Check your answer at $\omega = 0$.

D5-06

Find $x(t)$ given

$$X(j\omega) = \frac{j\omega + 6}{(j\omega + 1)(j\omega + 4)}.$$

a) Split X into partial fractions.

b) Give $x(t)$.

c) Check your answer at $\omega = 0$, and state $x(0^+)$.

D5-07

Find $x(t)$ given

$$X(j\omega) = \frac{j\omega + 3}{(j\omega + 1)^2(j\omega + 2)}.$$

a) Split X into partial fractions, using the repeated-pole rule at $j\omega = -1$.

b) Give $x(t)$.

c) Check your answer at $\omega = 0$ and at $t = 0^+$.

D5-08

A system is described by

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 6\frac{\mathrm{d}y}{\mathrm{d}t} + 8y = 2x(t).$$

a) Find the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$.

b) Split H into partial fractions and give the impulse response $h(t)$.

c) Check $H(0)$ against the area of $h(t)$.

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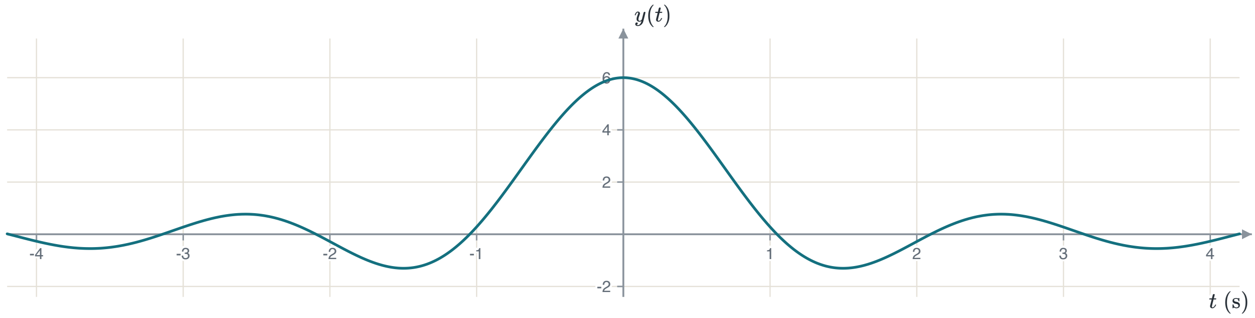
D5-09

Let

$$y(t) = \frac{6\sin(3t)}{3t} = \frac{2\sin(3t)}{t}.$$



a) Identify the known transform pair $y(t)$ is built from, using the unnormalised sinc, $\mathrm{sinc}(\theta) = \sin\theta/\theta$.

b) Apply duality to obtain $Y(j\omega)$, and sketch it.

c) Check $Y(0)$ against the area of $y(t)$.

D5-10

Let

$$y(t) = e^{-3t}u(t), \qquad z(t) = \frac{\mathrm{d}y}{\mathrm{d}t}.$$

a) State the differentiation property and use it to find $Z(j\omega)$ from $Y(j\omega)$, without differentiating in time.

b) Write $z(t)$ explicitly, using the product rule on $e^{-3t}u(t)$, and note the term that comes from the jump in y at $t = 0$.

c) Check $Z(0)$ against the area of $z(t)$.

D5-11

Let $p(t) = 1$ for $|t| < 1$, 0 elsewhere, with $P(j\omega) = 2\sin(\omega)/\omega$. Let

$$w(t) = p(3t - 6).$$

a) Write $w(t)$ in the form $p(at - b)$ and identify a and b.

b) Apply the scaling property, then the time-shift property, to find $W(j\omega)$ without integrating.

c) Check $W(0)$ against the area of $w(t)$.

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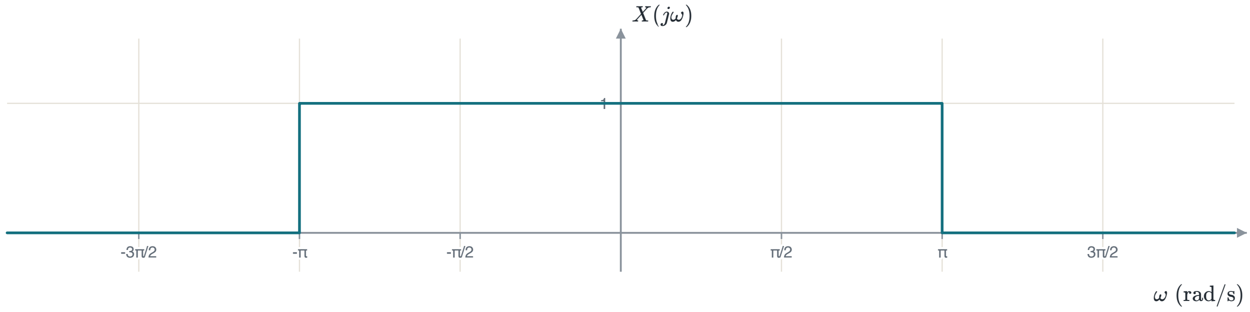
D5-12

Let $x(t) = \dfrac{\sin(\pi t)}{\pi t}$, whose transform is $X(j\omega) = 1$ on $|\omega| < \pi$, 0 elsewhere. Let

$$z(t) = e^{j3t}x(t).$$



a) State the frequency-shift property in the form used here, naming what is fixed and what is being multiplied.

b) Apply it to find $Z(j\omega)$ and sketch it, giving the band edges numerically.

c) Check $z(0)$ against $x(0)$, and again against $\dfrac{1}{2\pi}\displaystyle\int Z(j\omega)\,\mathrm{d}\omega$.

D5-13

Let $x(t) = 3e^{-2t}u(t)$.

a) Compute the total energy directly in the time domain.

b) Compute it again from $X(j\omega)$ using Parseval's relation.

D5-14

Let $y(t) = e^{-4|t|}$, whose transform is $Y(j\omega) = \dfrac{8}{16+\omega^2}$.

a) Compute the total energy of $y(t)$ in the time domain.

b) Use Parseval's relation to evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{(16+\omega^2)^2}$.

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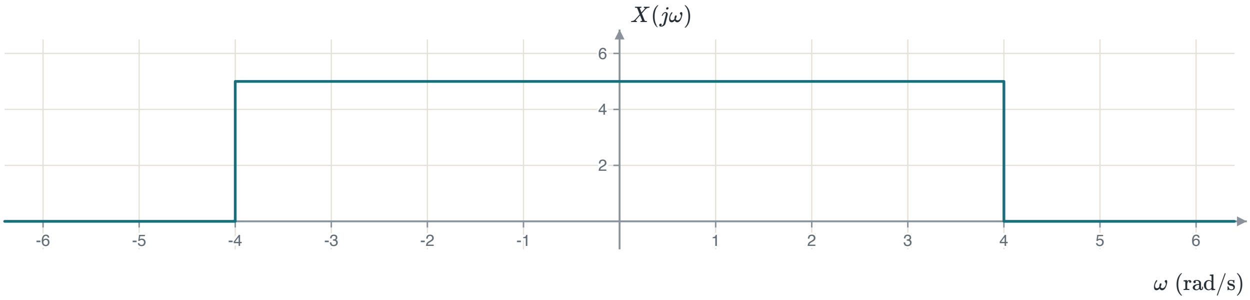
D5-15

A signal has $X(j\omega) = 5$ for $|\omega| < 4$, 0 elsewhere.



a) Compute the total energy of $x(t)$ using Parseval.

b) Find $x(t)$ from the ideal low-pass pair, and check $x(0)$ against $\dfrac{1}{2\pi}\displaystyle\int X(j\omega)\,\mathrm{d}\omega$.

c) Find the fraction of the total energy that lies in $|\omega| < 2$.

D5-16

Let $x_1(t) = 2e^{-3t}u(t)$ and $x_2(t) = 2e^{3t}u(-t) = x_1(-t)$.

a) Compute E_1, the energy of $x_1(t)$, in the time domain.

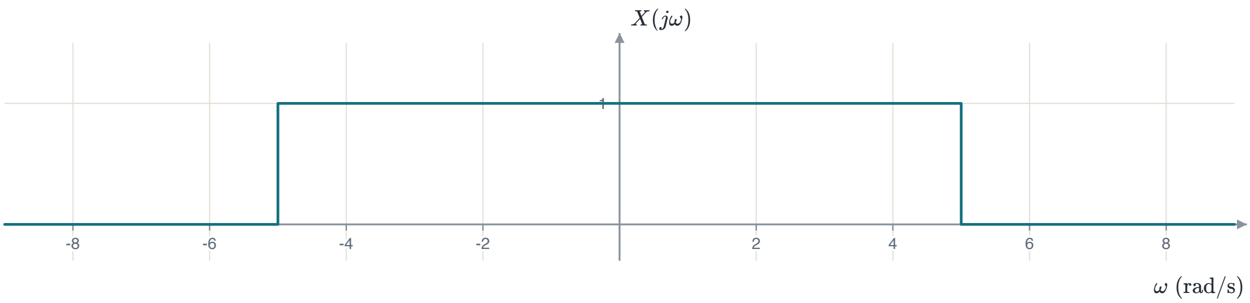
b) Find $X_2(j\omega)$ using the time-reversal property, and say what it implies about $|X_2(j\omega)|$ compared with $|X_1(j\omega)|$.

c) State E_2 without a new frequency integral, then confirm it directly in the time domain.

D5-17

A signal has $X(j\omega) = 1$ for $|\omega| < 5$, 0 elsewhere. It is modulated by a carrier:

$$z(t) = x(t)\cos(20t).$$



a) Give $Z(j\omega)$.

b) Sketch $Z(j\omega)$, showing the negative-frequency copy.

c) State the general non-overlap condition on the carrier frequency, and verify it holds here.

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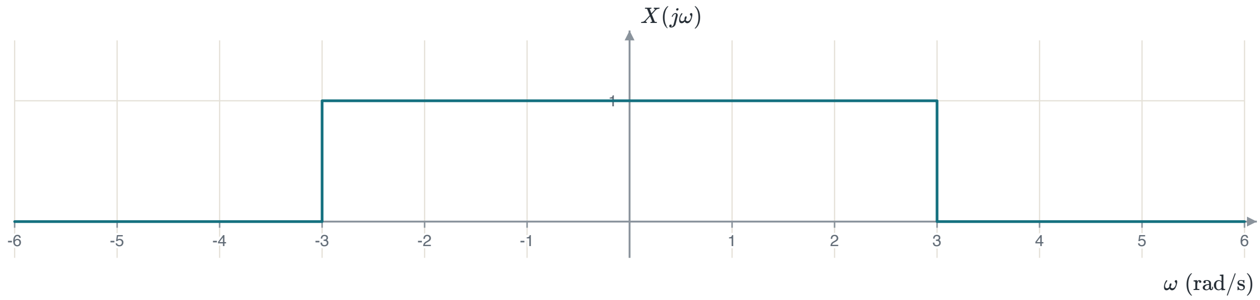
D5–18

A baseband signal has $X(j\omega) = 1$ for $|\omega| < 3$, 0 elsewhere. It is sent through the chain

$$x(t) \xrightarrow{\times\cos(10t)} z(t) \xrightarrow{\times\cos(10t)} w(t) \xrightarrow{H(j\omega)} y(t),$$

where the last stage is an ideal low-pass filter, $H(j\omega) = 2$ for $|\omega| < 5$, 0 elsewhere.



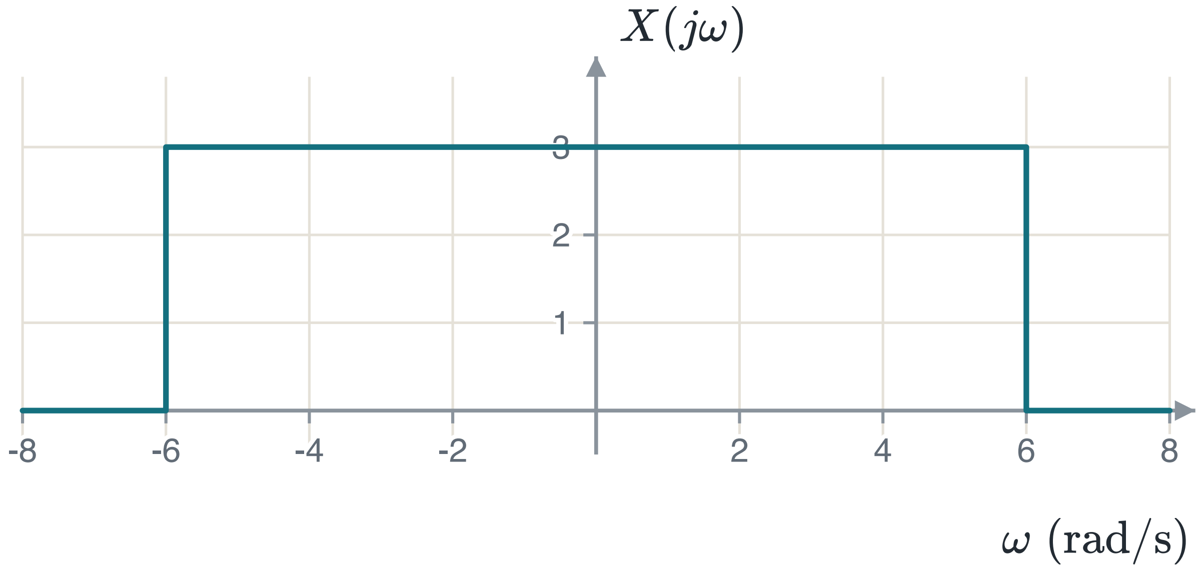
a) Give $Z(j\omega)$, the spectrum after the first multiplication, and its two bands.

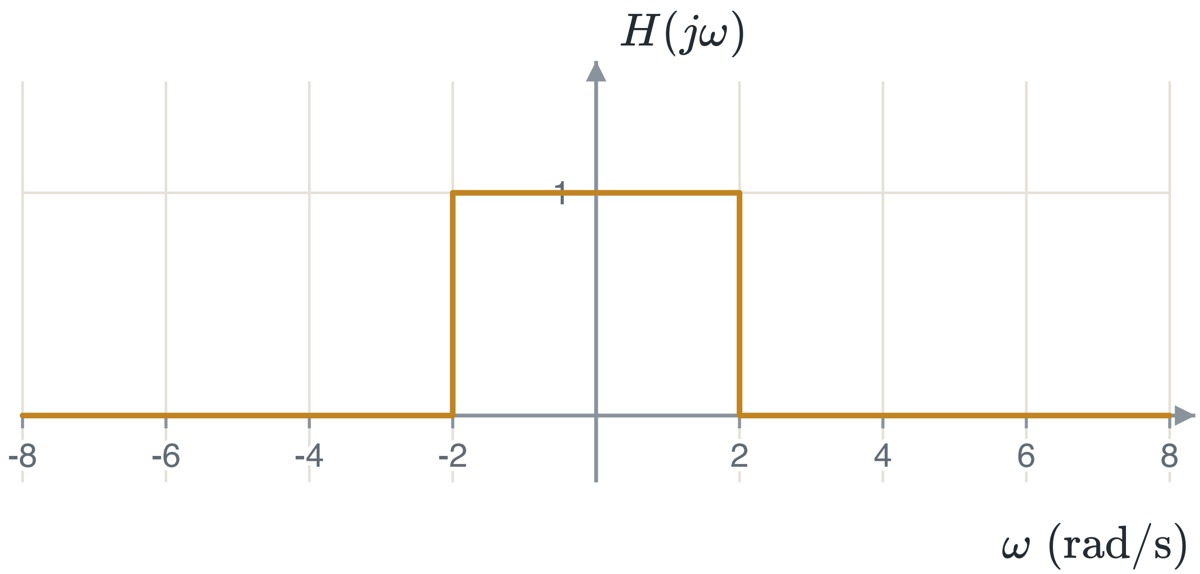
b) Give $W(j\omega)$, the spectrum after the second multiplication, before the filter.

c) Give $Y(j\omega)$ after the filter, and confirm it reproduces $X(j\omega)$ exactly.

D5-19

A signal has $X(j\omega) = 3$ for $|\omega| < 6$, 0 elsewhere. It is passed through an ideal low-pass filter, $H(j\omega) = 1$ for $|\omega| < 2$, 0 elsewhere.





a) Give $Y(j\omega) = X(j\omega)H(j\omega)$ and sketch it beside $X(j\omega)$.

b) Find $x(t)$ and $y(t)$ in closed form.

c) Compare $y(0)$ with $x(0)$, and relate the ratio to the two bandwidths.

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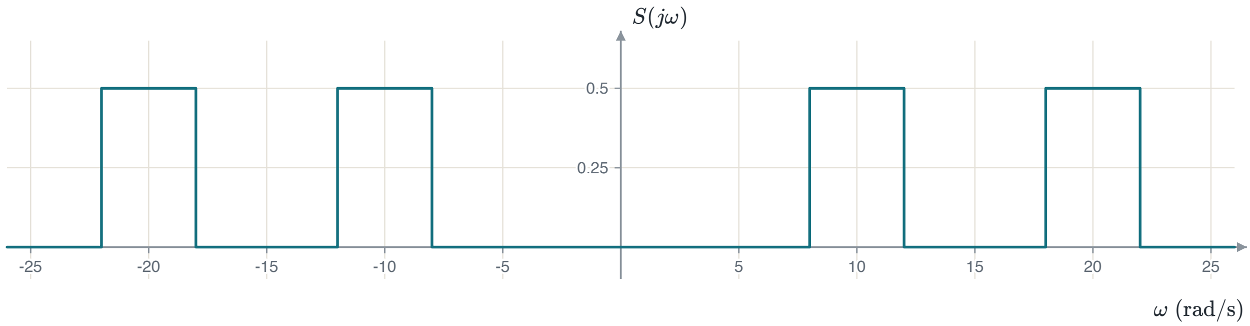
D5-20

Two baseband signals, each with $X_1(j\omega) = X_2(j\omega) = 1$ for $|\omega| < 2$, are sent on one channel by two carriers:

$$s(t) = x_1(t)\cos(10t) + x_2(t)\cos(20t).$$

A receiver recovers $x_2(t)$ by multiplying $s(t)$ by $\cos(20t)$ and then applying an ideal low-pass filter, $H(j\omega) = 2$ for $|\omega| < 2$, 0 elsewhere.



a) Give $S(j\omega)$, listing the centre and height of all four bands.

b) After multiplying by $\cos(20t)$ a second time, give the part of the resulting spectrum lying in $|\omega| < 2$, and say which baseband signal it came from.

c) Apply the filter and confirm the output equals $X_2(j\omega)$ exactly. State the nearest surviving band and how far it sits from the filter's cut-off.

D5-21

Determine the Fourier transforms of the following signals and plot their corresponding magnitude spectra.

a) $x(t) = \cos(2t)\,u(t)$.

b) $x(t) = u(t+3) - u(t-3)$.

c) $x(t) = e^{-t(2+j10\pi)}u(t)$.

D5-22

Please solve the following problems. *Hint: if* $x(t) \xrightarrow{\mathcal{F}} X(j\omega) = X(\omega)$, *then* $X(t) \xrightarrow{\mathcal{F}} 2\pi x(-\omega)$.

a) Determine the Fourier transform of $x(t) = e^{-3|t|}$.

b) Determine the Fourier transform of $y(t) = \dfrac{6}{9+t^2}$.

c) Calculate the total energy in $y(t)$, which is defined in part (b).

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D5-23

Please solve the following problems.

a) Determine the inverse Fourier transform of $X(j\omega) = \dfrac{j\omega + 7}{(j\omega)^2 + 5j\omega + 6}$.

b) Determine the Fourier transform of $x(t) = e^{-3(t+2)} \cos(4t)\, u(t+2)$.

c) Determine the Fourier transform of $x(t) = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k \delta(t-2k)$.

D5-24

Let

$$x(t) = \cos(t) + \cos(3t) + 2\cos(5t)$$

be an input to the LTI system which has the following impulse response,

$$h(t) = \pi \frac{\sin(2t)}{\pi t} \frac{\sin(4t)}{\pi t}.$$

Let $y(t) = x(t) * h(t)$ be the output signal.

a) Plot the Fourier transform of $x(t)$.

b) Plot the Fourier transform of $h(t)$.

c) Plot the Fourier transform of $y(t)$, and give $y(t)$.

D5-25

Let

$$x(t) = \frac{1 - e^{-j8\pi t}}{j2\pi t}$$

be an input to the following communication chain, where $\times$ shows the multiplication operator and the LTI system has the impulse response $h(t) = e^{-j3\pi t} \dfrac{\sin(2\pi t)}{\pi t}$. The input is multiplied by $\cos(6\pi t)$ to give $y(t)$, and $y(t)$ is filtered by $h(t)$ to give $z(t)$.

a) Plot the Fourier transform of $x(t)$. *Hint: can you write* $x(t)$ *as* $e^{jW_1 t} \dfrac{\sin(W_2 t)}{\pi t}$ *for some* W_1, W_2*?*

b) Plot the Fourier transform of $y(t)$.

c) Plot the Fourier transform of $z(t)$.

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D5-26

Consider the signal

$$x(t) = \left(\frac{\sin(3t)}{\pi t}\right)^2 \left(1 + e^{j12t}\right),$$

which is input to an LTI system with the impulse response $h(t) = \dfrac{\sin(5t)}{\pi t} e^{j5t}$. At the output of the LTI system, the $y(t)$ signal is observed.

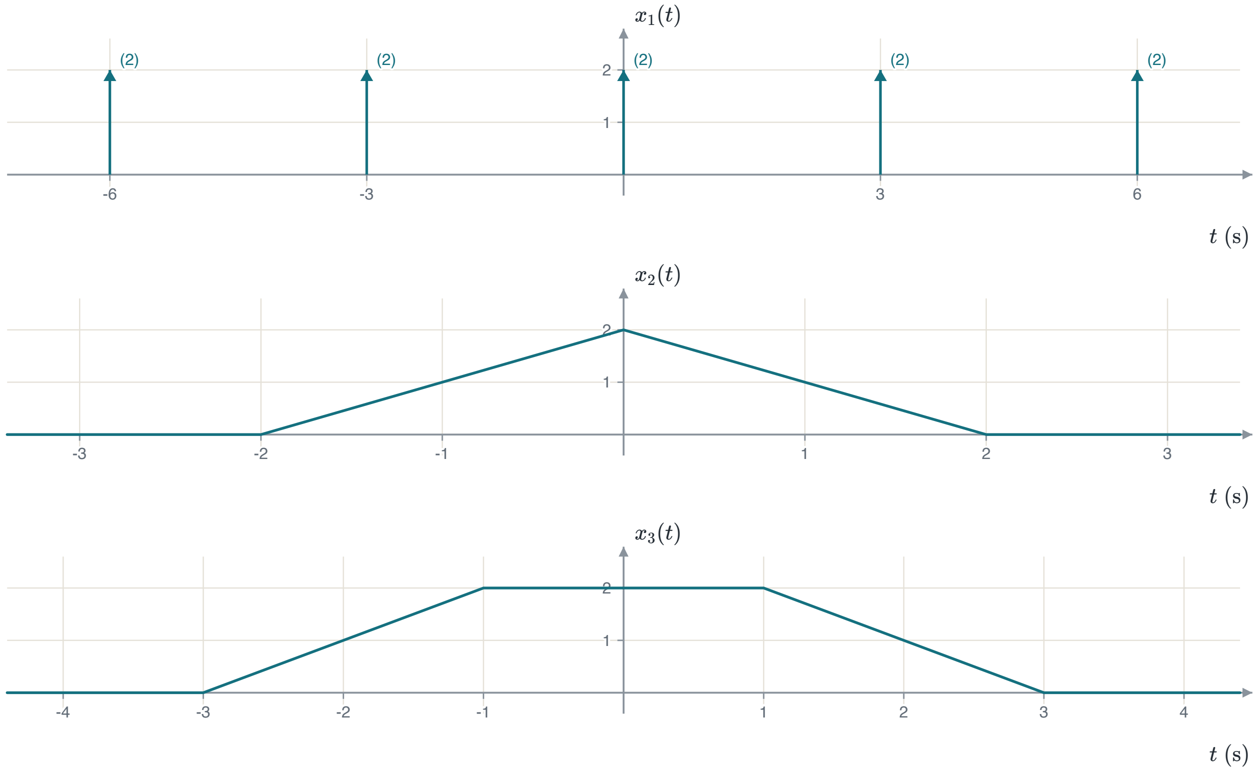
a) Plot the Fourier transform of $x(t)$.

b) Plot the Fourier transform of $h(t)$.

c) Plot the Fourier transform of $y(t)$.

D5-27

Consider the three signals plotted below, where $x_1(t)$ is periodic and its plot shows the areas under its impulses, while $x_2(t)$ and $x_3(t)$ are aperiodic.



a) Determine the Fourier transform of $x_1(t)$.

b) Determine the Fourier transform of $x_2(t)$.

c) Determine the Fourier transform of $x_3(t)$. *Hint: consider building it from rectangles.*

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D5-28

Let $x(t) = 1 + 3\cos(2\pi t)$ be a periodic signal, and let $s(t) = x(t)c(t)$ where $c(t) = \cos(10\pi t)$.

a) Determine the Fourier transform of $x(t)$.

b) Plot the Fourier transform of $s(t)$.

c) State the smallest carrier frequency for which the shifted copies would not overlap, and say what happens below it.

D5-29

Determine the Fourier transforms of the following signals.

a) $x(t) = e^{-3t}u(t-2)$.

b) $x(t) = t\,e^{-3t}u(t)$.

c) $x(t) = [u(t+2) - u(t-2)] * [u(t+2) - u(t-2)]$, where $*$ is the convolution operator.

D5-30

Let $x(t) = \dfrac{\sin(4t)}{\pi t}$, and let $y(t) = x(t)\cos(6t)$.

a) Plot the Fourier transform of $x(t)$ and calculate its total energy.

b) Plot the Fourier transform of $y(t)$.

c) Calculate the total energy of $y(t)$ and compare it with that of $x(t)$.

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MODULE

6

Discrete-Time Fourier Transform

30 questions on discrete-time fourier transform. Write your reasoning in the space under each one.

D6–01

Find the discrete-time Fourier transform of

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n].$$

a) Compute $X(e^{j\omega})$ in closed form, and state the condition it needs to converge.

b) Evaluate $\left|X(e^{j\omega})\right|$ at $\omega = 0$ and $\omega = \pi$.

c) Plot $\left|X(e^{j\omega})\right|$ over one period.

D6–02

Find the discrete-time Fourier transform of

$$x[n] = 3\left(\tfrac{1}{4}\right)^{|n|}.$$

a) Split the sum at $n = 0$ and derive $X(e^{j\omega})$ in closed form.

b) Evaluate $X(e^{j0})$ and $X(e^{j\pi})$.

c) State whether $X(e^{j\omega})$ is real for every ω, and why.

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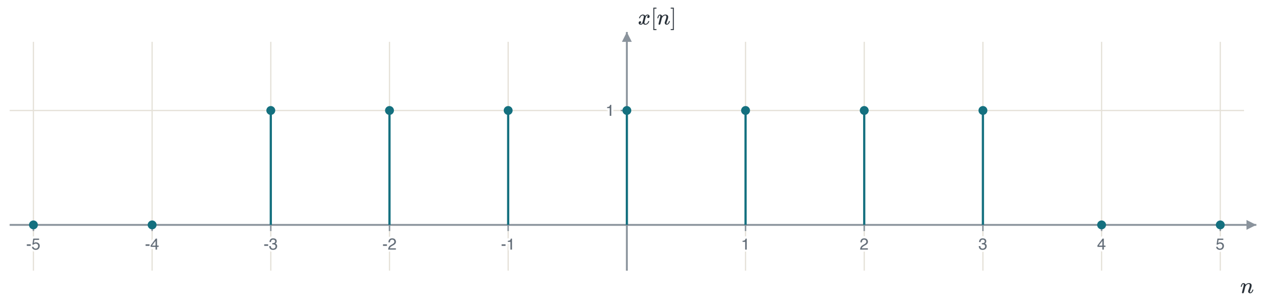
D6-03

The sequence $x[n]$ shown below equals 1 for $-3 \le n \le 3$ and is zero elsewhere.



a) State the support of $x[n]$ and give $X(e^{j\omega})$ as a ratio of sines.

b) Give $X(e^{j0})$ and locate the zeros of X inside $-\pi \le \omega \le \pi$.

c) Plot $\left|X(e^{j\omega})\right|$ over one period, marking the zeros.

D6-04

Find the discrete-time Fourier transform of

$$x[n] = 3\left(\tfrac{1}{3}\right)^{n-1} u[n-1].$$

a) Compute $X(e^{j\omega})$ directly from the analysis sum.

b) Give $X(e^{j\omega})$ a second way, using the time-shift property on $w[n] = 3\left(\tfrac{1}{3}\right)^{n} u[n]$, and confirm the two routes agree.

c) Evaluate $X(e^{j0})$ and check it against the sum of the sequence.

D6-05

Plot the discrete-time Fourier transform of

$$x[n] = 4\cos\left(\tfrac{\pi}{3}n\right)$$

on $-\pi \le \omega \le \pi$.

a) Give $X(e^{j\omega})$ as a sum of impulses on one period.

b) Mark the impulse weights and locations on a plot.

c) Confirm the weights by synthesising $x[0]$ from the two impulses.

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D6-06

Plot the discrete-time Fourier transform of

$$x[n] = 2 + 3\cos\left(\tfrac{2\pi}{5}n\right)$$

on $-\pi \le \omega \le \pi$.

a) Give $X(e^{j\omega})$ as a sum of impulses on one period.

b) State the weight of the impulse at $\omega = 0$, and say why it differs from the weight the cosine term contributes at either of its two frequencies.

c) Plot the spectrum, marking every weight.

D6-07

Find the discrete-time Fourier transform of

$$x[n] = 5\sin\left(\tfrac{3\pi}{4}n\right).$$

a) Derive the impulse pair for $\sin(\omega_0 n)$ from the impulse pair for $e^{j\omega_0 n}$, using Euler's relation.

b) Give $X(e^{j\omega})$ for this $x[n]$.

c) Check the result by synthesising $x[1]$.

D6-08

Plot the discrete-time Fourier transform of

$$x[n] = 2\cos\left(\tfrac{3\pi}{2}n\right) + \cos\left(\tfrac{9\pi}{4}n\right)$$

on $-\pi \le \omega \le \pi$.

a) Reduce each frequency into $-\pi \le \omega \le \pi$, and confirm the reduction leaves $x[n]$ unchanged at $n = 2$.

b) Give $X(e^{j\omega})$ as a sum of impulses over one period.

c) Plot the spectrum, and check the total weight against $x[0]$.

D6-09

A discrete-time LTI system has impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$ and input $x[n] = \cos\left(\frac{\pi}{2}n\right)$.

a) Give $H(e^{j\omega})$.

b) Evaluate H at $\omega = \frac{\pi}{2}$, and give $y[n]$ in amplitude-and-phase form.

c) Check $\left|H(e^{j\pi/2})\right|$ using $\left|H(e^{j\omega})\right|^2 = H(e^{j\omega})H^*(e^{j\omega})$.

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D6-10

A discrete-time LTI system has impulse response

$$h[n] = \delta[n] - \delta[n-1].$$

a) Give $H(e^{j\omega})$, and show that $\left|H(e^{j\omega})\right| = 2\,|\sin(\omega/2)|$.

b) Give $|H|$ at $\omega = 0$ and $\omega = \pi$, and say what kind of filter this is.

c) Plot $\left|H(e^{j\omega})\right|$ over one period.

D6-11

A discrete-time LTI system has impulse response $h[n] = \left(\frac{1}{4}\right)^n u[n]$ and input $x[n] = \left(\frac{1}{2}\right)^n u[n]$.

a) Give $X(e^{j\omega})$, $H(e^{j\omega})$ and $Y(e^{j\omega})$.

b) Recover $y[n]$ by partial fractions.

c) Check $y[0]$ and $y[1]$ directly against the convolution sum.

D6-12

A two-tap averaging filter has impulse response

$$h[n] = \tfrac{1}{2}\delta[n] + \tfrac{1}{2}\delta[n-1],$$

and its input is $x[n] = \cos\left(\frac{\pi}{3}n\right)$.

a) Write $H(e^{j\omega}) = e^{-j\omega/2}\cos(\omega/2)$, and evaluate it at $\omega = \frac{\pi}{3}$.

b) Give $y[n]$ in amplitude-and-phase form.

c) Give $Y(e^{j\omega})$ as a pair of weighted impulses, and plot $\left|Y(e^{j\omega})\right|$.

D6-13

Find $x[n]$ when

$$X(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{3}, \\ 0, & \frac{\pi}{3} < |\omega| \le \pi, \end{cases}$$

repeated with period 2π.

a) Compute $x[n]$ from the synthesis integral, for $n \neq 0$.

b) Give $x[0]$ two ways: as the limit of your part (a) formula, and as the mean of the spectrum over one period.

c) Plot $x[n]$ for $-8 \le n \le 8$.

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D6-14

Find $x[n]$ when

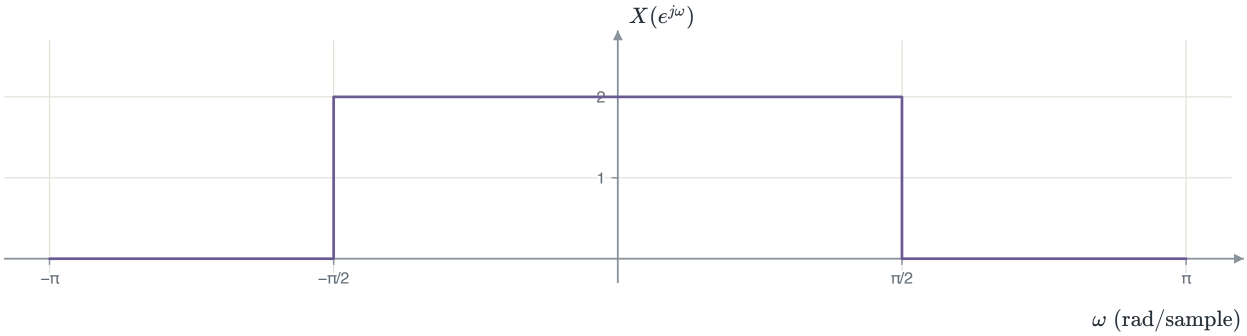
$$X(e^{j\omega}) = 4 - e^{-j\omega} + 2e^{-j3\omega}.$$

a) Give $x[n]$.

b) State the support of $x[n]$ and give $\sum_n x[n]$.

D6-15

The spectrum $X(e^{j\omega})$ shown below is one period of a periodic function.



a) Read the height and the cut-off frequency of $X(e^{j\omega})$ from the figure, and write it as a formula.

b) Compute $x[n]$ for $n \neq 0$ from the synthesis integral.

c) Give $x[0]$, and check it against the mean of the spectrum read directly from the figure.

D6-16

Find $x[n]$ when

$$X(e^{j\omega}) = e^{-j\omega}\left(1 + \cos\omega\right).$$

a) Expand $X(e^{j\omega})$ into a sum of terms $e^{-j\omega n_0}$.

b) Read off $x[n]$ from the expansion.

c) Confirm the result at $\omega = 0$ and at $\omega = \pi$, against the original form.

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D6-17

Let $x[n]$ be a real sequence with transform $X(e^{j\omega}) = \sum_n x[n]e^{-j\omega n}$.

a) Prove that $X(e^{-j\omega}) = X^*(e^{j\omega})$, directly from the analysis sum.

b) For $x[n] = \left(\frac{1}{3}\right)^n u[n]$, give $X(e^{j\pi/3})$ in the form $p + jq$, and give $X(e^{-j\pi/3})$ the same way by direct substitution.

c) Confirm that the two values of part (b) are complex conjugates, and say why one half of a period is enough to describe the spectrum of a real sequence but not of a complex one.

D6-18

Let $x[n] = \delta[n+1] - \delta[n-1]$.

a) Give $X(e^{j\omega})$, and confirm $X(e^{j0}) = 0$.

b) Apply the differencing property to find $W(e^{j\omega})$ for $w[n] = x[n] - x[n-1]$, and confirm it against $w[n]$ written out directly.

c) Apply the accumulation property to find $Y(e^{j\omega})$ for $y[n] = \sum_{k=-\infty}^{n} x[k]$, and confirm it against $y[n]$ written out directly.

D6-19

Let $x[n] = 2\left(\frac{1}{2}\right)^n u[n]$.

a) Compute the energy of $x[n]$ directly, in the time domain.

b) Write Parseval's relation for $X(e^{j\omega})$, and state what $\left|X(e^{j\omega})\right|^2$ represents.

c) Using $\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\mathrm{d}\omega}{b - \cos\omega} = \frac{1}{\sqrt{b^2 - 1}}$ for $b > 1$, evaluate the frequency-side integral and confirm it matches part (a).

D6-20

Let $x[n]$ be an arbitrary sequence with transform $X(e^{j\omega}) = \sum_n x[n]e^{-j\omega n}$, convergent for every ω.

a) Prove that $X\left(e^{j(\omega+2\pi)}\right) = X(e^{j\omega})$, directly from the analysis sum.

b) Two signal generators produce the sequences $\cos(2.3\,n)$ and $\cos((2.3 - 2\pi)n)$. Determine whether the two generators produce the same sequence, and say what part (a) implies about telling them apart from their output alone.

c) State the highest discrete-time frequency that exists, and explain in one sentence why continuous time has no such bound.

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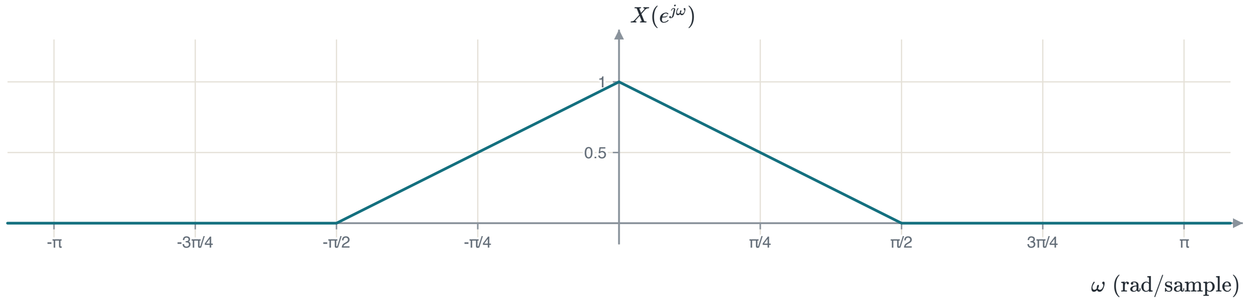
D6–21

Determine the discrete-time Fourier transforms of the following signals. In parts (b) and (c), $\mathcal{F}\{x[n]\}$ is the triangular spectrum plotted below: it equals 1 at $\omega = 0$ and falls linearly to zero at $\omega = \pm\frac{\pi}{2}$.



a) $z[n] = u[-n+4] - u[-n-1]$.

b) $y[n] = x[n]p[n]$, where $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$.

c) $y[n] = x[n]p[n]$, where $p[n] = (-1)^n$.

D6–22

Please solve the following problems, giving every answer in $-\pi \le \omega \le \pi$.

a) Determine the DTFT of $x[n] = \left(\frac{1}{3}\right)^{|n|}$.

b) Plot the DTFT of $x[n] = \cos\left(\frac{5\pi}{2}n\right) + 2\cos\left(\frac{4\pi}{3}n\right)$.

c) Plot the DTFT of $x[n] = \dfrac{\cos(\pi n)}{\pi n}\sin\left(\frac{\pi}{3}n\right)$.

D6–23

Consider the discrete-time signal

$$x[n] = \frac{\sin\left(\frac{3\pi}{5}n\right) + 2\sin\left(\frac{\pi}{5}n\right)}{\pi n},$$

which is input to an LTI system with the impulse response

$$h[n] = 2\,\frac{\cos(\pi n)}{\pi n}\sin\left(\frac{4\pi}{5}n\right).$$

At the output of the LTI system, the $y[n]$ signal is observed.

a) Plot the DTFT of $x[n]$ in $-\pi \le \omega \le \pi$.

b) Plot the DTFT of $h[n]$ in $-\pi \le \omega \le \pi$.

c) Plot the DTFT of $y[n]$ in $-\pi \le \omega \le \pi$.

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D6–24

Determine the discrete-time Fourier transforms of the following signals.

a) $x[n] = \left(\frac{1}{2}\right)^n u[n-1]$.

b) $x[n] = u[n+2] - u[n-3]$.

c) $x[n] = \left(\frac{1}{2}\right)^{|n|}$.

D6–25

A sequence $x[n]$ has the discrete-time Fourier transform $X(e^{j\omega})$. Express the DTFT of each of the following in terms of $X(e^{j\omega})$.

a) $y[n] = x[n-3]$.

b) $y[n] = (-1)^n x[n]$.

c) $y[n] = x[n] - x[n-1]$, and state the value of Y at $\omega = 0$.

D6–26

Determine the inverse discrete-time Fourier transform of each of the following spectra, each specified over $-\pi \leq \omega \leq \pi$ and repeating with period 2π.

a) $X(e^{j\omega}) = 1$ for $|\omega| < \frac{\pi}{4}$ and zero otherwise.

b) $X(e^{j\omega}) = \cos(2\omega)$.

c) $X(e^{j\omega}) = \dfrac{1}{\left(1 - \frac{1}{3}e^{-j\omega}\right)^2}$.

D6–27

Let $x[n] = \dfrac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}$.

a) Give $X(e^{j\omega})$ over $-\pi \leq \omega \leq \pi$ and state $x[0]$.

b) Calculate the total energy of $x[n]$ using Parseval.

c) Let $y[n] = (-1)^n x[n]$. Give $Y(e^{j\omega})$ and its energy.

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D6–28

An LTI system has the impulse response $h[n] = \left(\frac{1}{3}\right)^n u[n]$ and is driven by the input $x[n] = \left(\frac{1}{2}\right)^n u[n]$.

a) Give $X(e^{j\omega})$ and $H(e^{j\omega})$.

b) Give $Y(e^{j\omega})$ and find $y[n]$ by partial fractions.

c) Verify $y[0]$ and $y[1]$ against the convolution sum.

D6–29

Two filters are defined by

$$h_1[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}, \qquad h_2[n] = \delta[n] - \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}.$$

a) Plot $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ over $-\pi \leq \omega \leq \pi$.

b) Determine the impulse response of the parallel connection $h_1[n] + h_2[n]$ and its frequency response.

c) Determine the frequency response of the series connection $h_1[n] * h_2[n]$, and say what the connection does.

D6–30

Let $p[n] = \sum_{k=-\infty}^{\infty} \delta[n - 5k]$ be a periodic impulse train, and let $x[n]$ be a sequence whose DTFT is 1 for $|\omega| < \frac{\pi}{5}$ and zero elsewhere in $-\pi \leq \omega \leq \pi$.

a) Determine the DTFT of $p[n]$.

b) Determine and plot the DTFT of $y[n] = x[n]p[n]$.

c) State whether $x[n]$ can be recovered from $y[n]$ by an ideal low-pass filter, and give the filter if it can.

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MODULE

7

Sampling and Aliasing

30 questions on sampling and aliasing. Write your reasoning in the space under each one.

D7–01

Find the Nyquist rate for

$$x(t) = \cos(40\pi t) + \cos(90\pi t).$$

a) List the frequencies present and give ω_M.

b) Give the Nyquist rate in rad/s and in hertz, and the largest usable sampling period.

D7–02

Find the Nyquist rate for

$$x(t) = \cos(30\pi t)\cos(80\pi t).$$

a) Rewrite $x(t)$ as a sum of cosines.

b) Give ω_M and the Nyquist rate, in rad/s and in hertz.

D7–03

Find the Nyquist rate for

$$x(t) = \sin(50\pi t)\cos(30\pi t).$$

a) Rewrite $x(t)$ as a sum of sines, using $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$.

b) Give ω_M and the Nyquist rate, in rad/s and in hertz.

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D7-04

Let

$$x(t) = \cos(20\pi t) + \cos(70\pi t), \qquad y(t) = x(t) + x^2(t).$$

a) List every frequency present in $y(t)$.

b) Give ω_M and the Nyquist rate for $y(t)$, in rad/s and in hertz.

D7-05

Find the Nyquist rate for

$$x(t) = \frac{\sin(300\pi t)}{\pi t}.$$

a) Give $X(j\omega)$.

b) Give ω_M and the Nyquist rate, in rad/s and in hertz.

D7-06

Find the Nyquist rate for

$$x(t) = \left[\frac{\sin(100\pi t)}{\pi t}\right]^2.$$

a) Describe $X(j\omega)$: its shape, its peak, and its edges.

b) Give ω_M and the Nyquist rate, in rad/s and in hertz.

c) Sketch $X(j\omega)$.

D7-07

Find the Nyquist rate for

$$x(t) = \frac{\sin(180\pi t)}{\pi t} + \frac{\sin(500\pi t)}{\pi t}.$$

a) Describe $X(j\omega)$ and sketch it.

b) Give ω_M and the Nyquist rate, and say which term determines it.

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D7-08

Find the Nyquist rate for

$$x(t) = \left[\frac{\sin(140\pi t)}{\pi t}\right]^2 + \frac{\sin(90\pi t)}{\pi t}.$$

a) Give the bandwidth of each of the two terms separately.

b) Give ω_M and the Nyquist rate for the sum, in rad/s and in hertz.

D7-09

Two signals are band-limited: $X_1(j\omega) = 0$ for $|\omega| > 70\pi$ and $X_2(j\omega) = 0$ for $|\omega| > 190\pi$. Define

$$y(t) = x_1(t) * x_2(t).$$

a) Give the band-limit of $y(t)$.

b) Give the Nyquist rate for $y(t)$, and justify the answer in one sentence.

D7-10

With the same two signals — $X_1(j\omega) = 0$ for $|\omega| > 70\pi$ and $X_2(j\omega) = 0$ for $|\omega| > 190\pi$ — define

$$z(t) = x_1(t)\, x_2(t).$$

a) Give the band-limit of $z(t)$.

b) Give the Nyquist rate for $z(t)$, and contrast the answer with the convolution case.

D7-11

Let

$$z(t) = \frac{\sin(60\pi t)}{\pi t} \cos(400\pi t).$$

a) Describe $Z(j\omega)$, giving the edges of every band it occupies.

b) Give ω_M and the Nyquist rate, in rad/s and in hertz.

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D7-12

Two signals are band-limited: $X_1(j\omega) = 0$ for $|\omega| > 50\pi$ and $X_2(j\omega) = 0$ for $|\omega| > 130\pi$. Their product $w(t) = x_1(t)x_2(t)$ is to be sampled and reconstructed with a filter whose transition band is $\omega_g = 20\pi$ rad/s wide.

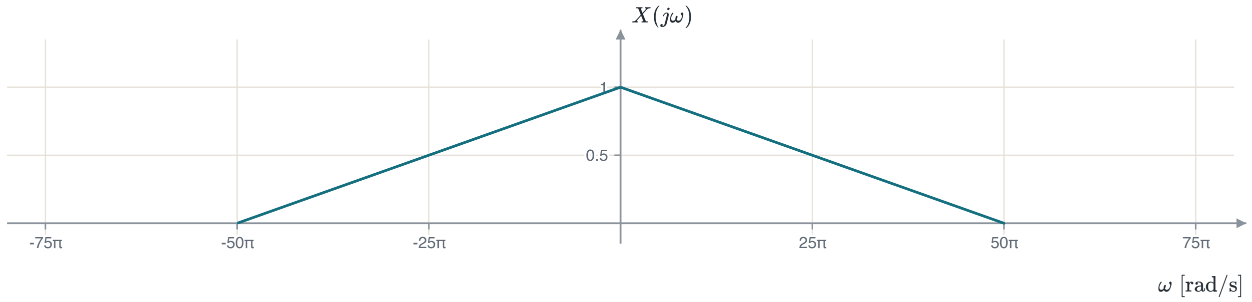
a) Give ω_M for $w(t)$.

b) Give the minimum sampling rate when an ideal reconstruction filter is assumed.

c) Give the minimum sampling rate that leaves room for the stated guard band.

D7-13

A signal has the triangular spectrum shown below, with $X(j\omega) = 0$ for $|\omega| > 50\pi$ rad/s. It is sampled with period $T = 1/300$ s.



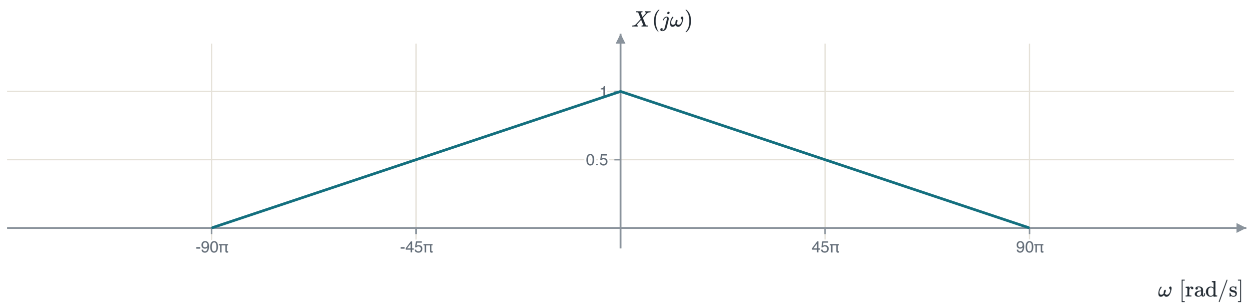
a) Read ω_M off the figure.

b) Give ω_s and the guard band $\omega_s - 2\omega_M$.

c) Sketch $X_p(j\omega)$ for $k = -1, 0, 1$, labelling both positive and negative frequencies.

D7-14

A signal has the triangular spectrum shown below, with $X(j\omega) = 0$ for $|\omega| > 90\pi$ rad/s. It is sampled at $\omega_s = 140\pi$ rad/s.



a) Confirm that this is undersampling.

b) Sketch the baseband together with the neighbouring copies, shading the overlap.

c) Give the width of the overlap on each side.

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D7-15

The signal $x(t) = \cos(2\pi \cdot 450t)$ is sampled at $f_s = 800$ Hz.

a) State whether the sampling theorem is satisfied.

b) Give the apparent frequency of the sampled sequence.

c) Sketch the spectral lines involved: the original frequency and the apparent one.

D7-16

The signal

$$x(t) = \cos(2\pi \cdot 150t) + \cos(2\pi \cdot 350t)$$

is sampled at $f_s = 500$ Hz.

a) Give the apparent frequency of each component.

b) Show that the two components alias onto the same apparent frequency, and simplify the sampled sequence.

c) Besides 350 Hz, name one further frequency above 500 Hz whose samples at this rate coincide with those of the 150 Hz component.

D7-17

A signal is sampled with period $T = 0.4$ ms and reconstructed with an ideal filter of cutoff $\omega_c = \pi/T$. The only non-zero samples are $x(0) = 3$ and $x(T) = -2$.

a) Write $h_{LP}(t)$ in closed form, using the unnormalised sinc convention $\text{sinc}(\theta) = \sin\theta/\theta$.

b) Write $x_r(t)$ as a sum of two shifted kernels.

c) Evaluate $x_r(T)$ directly from the formula, and confirm it matches the given sample.

d) Evaluate $x_r(1.5T)$.

D7-18

A zero-order hold has sampling period $T = 0.25$ ms.

a) Give $H_0(j\omega)$ in closed form.

b) Evaluate $H_0(0)$ and compare it with the ideal reconstruction filter's gain.

c) Evaluate $|H_0(j\pi/T)|$, and give it as a fraction of the ideal gain.

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D7-19

The signal $x(t) = \cos(50\pi t) + \cos(140\pi t)$ is sampled at $f_s = 100$ Hz and reconstructed with cutoff $\omega_c = \omega_s/2$.

a) Without any filter ahead of the sampler, give the reconstructed signal and the error $e(t) = x(t) - x_r(t)$.

b) With a lowpass filter of cutoff $\omega_s/2$ placed ahead of the sampler, give the reconstructed signal and the error.

c) Compute the time-averaged mean-square error in both cases and compare them.

D7-20

Let

$$x(t) = 5 + 3\cos(1200\pi t) + 6\sin(3600\pi t),$$

sampled exactly at its own Nyquist rate.

a) Give ω_M and confirm $\omega_s = 2\omega_M = 7200\pi$ rad/s, so $T = 1/3600$ s.

b) Show directly from the samples that the $\sin(3600\pi t)$ term contributes nothing.

c) Show the same result in the frequency domain, from the weights the baseband and the $k = 1$ copy carry at $\omega = 3600\pi$.

d) A guard band of $\omega_g = 400\pi$ rad/s is now required. Give the new minimum sampling rate.

D7-21

Determine the Nyquist sampling rate for each of the following signals.

a) $y(t) = x(t) + x^2(t)$, where $x(t) = \cos(80\pi t) + \cos(160\pi t)$.

b) $y(t) = \dfrac{\sin(80\pi t)}{\pi t}\left[1 + e^{j200\pi t}\right]$.

c) $y(t) = \dfrac{\sin(80\pi t)}{\pi t} * \dfrac{\sin(240\pi t)}{\pi t}$, where $*$ is the convolution operator.

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D7–22

Plot the Fourier transforms of each of the signals given below and determine the Nyquist sampling frequencies (in rad/s) for each of these signals.

a) $x(t) = \dfrac{\sin(30\pi t)}{\pi t} + \cos(40\pi t)$.

b) $x(t) = \cos(30\pi t)\cos(70\pi t)$.

c) $x(t) = \dfrac{\sin(30\pi t)}{\pi t}\,\dfrac{\sin(60\pi t)}{\pi t}$.

D7–23

Plot the Fourier transforms of each of the signals given below, and determine the Nyquist sampling frequencies (in rad/s) for each of these signals while considering a guard band of $\omega_g = 80\pi$ rad/s.

a) $x(t) = \dfrac{\sin(120\pi t)}{\pi t} + \dfrac{\sin(240\pi t)}{\pi t}$.

b) $h(t) = x(t)\cos(160\pi t)$.

c) $y(t) = x(t) * h(t)$, where $*$ is the convolution operator.

D7–24

A signal $x(t) = 3\cos(200\pi t) + 2\sin(500\pi t)$ is sampled at $f_s = 400$ Hz.

a) Determine the Nyquist rate of $x(t)$ in hertz.

b) State which components alias at $f_s = 400$ Hz, and give the apparent frequency of each.

c) Write the continuous-time signal that the samples appear to have come from.

D7–25

A band-limited signal has $X(j\omega) = 0$ for $|\omega| > 1000\pi$ and is sampled with an impulse train of period $T = \frac{1}{1500}$ s.

a) Give the sampling frequency ω_s and say whether the Nyquist condition is met.

b) Describe the spectrum of the sampled signal $x_p(t)$ and the width of the gap between copies.

c) Give an ideal reconstruction filter that recovers $x(t)$ exactly, with its gain and a valid cut-off.

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D7-26

An audio channel is sampled at $f_s = 8$ kHz with no anti-aliasing filter. Four pure tones are applied in turn: 3 kHz, 5 kHz, 9 kHz and 12 kHz.

a) Give the folding frequency and say which tones are sampled correctly.

b) Give the apparent frequency of each tone after sampling.

c) Two of the four tones become indistinguishable. Identify them and say what filter placed where would have prevented it.

D7-27

Determine the Nyquist sampling rate for each of the following signals.

a) $y(t) = \cos^2(300\pi t)$.

b) $y(t) = \dfrac{\sin(100\pi t)}{\pi t}\cos(400\pi t)$.

c) $y(t) = \left[\dfrac{\sin(150\pi t)}{\pi t}\right]^2$.

D7-28

A signal $x(t)$ is band-limited to $|\omega| < 600\pi$ and is sampled at $\omega_s = 2000\pi$ rad/s to give the sequence $x[n] = x(nT)$. Every second sample is then discarded, giving $v[n] = x[2n]$.

a) Give the effective sampling rate after the samples are discarded.

b) State whether $x(t)$ can still be recovered from $v[n]$, with reasons.

c) Give the largest bandwidth a signal could have and still survive the same treatment.

D7-29

Determine the Nyquist sampling rate for each of the following signals, where $\omega_g = 0$ (no guard band is required).

a) $y(t) = x_1(t)x_2(t)$, where x_1 is band-limited to 200π and x_2 to 500π.

b) $y(t) = x_1(t) + x_2(t)$, with the same two signals.

c) $y(t) = x_1(t) * x_2(t)$, with the same two signals.

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D7-30

A signal $x(t) = \dfrac{\sin(400\pi t)}{\pi t}$ is sampled at $\omega_s = 1000\pi$ rad/s, and the samples are passed through an ideal reconstruction filter of gain T and cut-off $\omega_c = 500\pi$.

a) Give $X(j\omega)$ and the Nyquist rate of $x(t)$.

b) Describe the spectrum after sampling and say whether the copies overlap.

c) Give the output of the reconstruction filter, and repeat the three parts for $\omega_s = 600\pi$.

Signals and Systems · Student Workbook

VERSION	DATE	CHANGES
v0	23 September 2026	First edition.

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