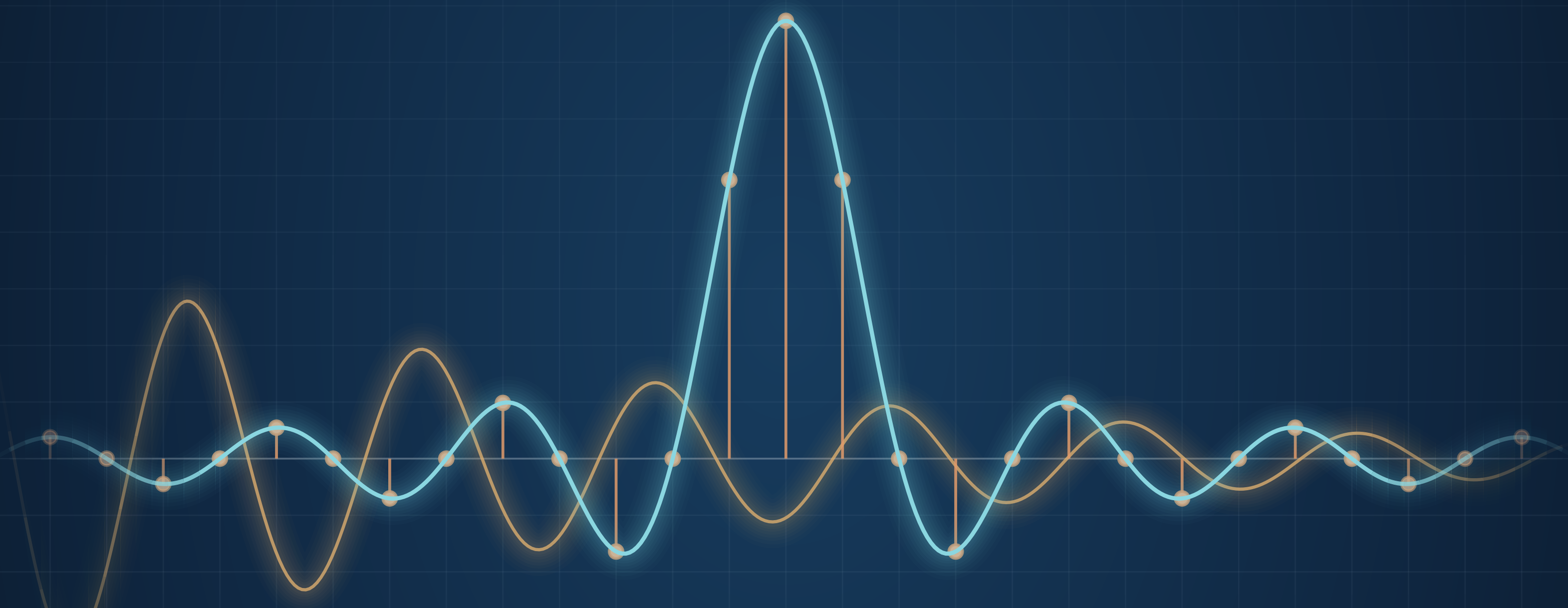
SIGNALS AND SYSTEMS

Signals, Systems and Frequency-Domain Analysis

Formula and Notation Reference



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CONVENTIONS · FORMULAS · NOTATION

Contents

The conventions used throughout the course, every formula it establishes, and every symbol it defines. Nothing here is derived; the derivations are in the lecture notes.

How to read it

Part 1 states the conventions and the two transform pairs. Part 2 is the summary of formulas, in the order the course establishes them. Part 3 defines every symbol. Nothing here is derived: where a result needs an argument, the argument is in the lecture notes chapter named beside it.

PART

1

Conventions

These hold everywhere in the course, without local variation. Where a result depends on one of them, it is restated at that point rather than assumed.

CONVENTION	STATEMENT
Energy and power	Normalised: the resistance is taken as $1\ \Omega$, so instantaneous power is $\lvert x\rvert^2$ and energy is its integral or sum.
Imaginary unit	j, with $j^2 = -1$.
Angular frequency	ω in rad/s in continuous time and rad/sample in discrete time. A frequency in hertz is written out as such, and $\omega = 2\pi f$.
Brackets	Round brackets for continuous time, $x(t)$. Square brackets for discrete time, $x[n]$, where n is an integer.
Convolution	$*$. It applies only to systems that are both linear and time invariant.
sinc	Unnormalised, $\mathrm{sinc}(\theta) = \sin\theta/\theta$, with zeros at $\theta = \pm k\pi$. The convention is restated wherever sinc is used.
Sampling	$\omega_s = 2\pi/T$ rad/s and $f_s = 1/T$ Hz. The sampling theorem is stated with a strict inequality, $\omega_s > 2\omega_M$.

THE CONTINUOUS-TIME FOURIER TRANSFORM PAIR

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

The first is the analysis equation and the second the synthesis equation. The factor $\frac{1}{2\pi}$ belongs to synthesis.

THE DISCRETE-TIME FOURIER TRANSFORM PAIR

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

The synthesis integral runs over one period, because the spectrum repeats every 2π.

PART

2

Summary of formulas

Everything the course establishes, in the order it establishes it.

A.1 Energy and power

Energy and power in continuous and discrete time.

QUANTITY	CONTINUOUS TIME	DISCRETE TIME
Total energy	$E_\infty = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \mathrm{d}t$	$E_\infty = \sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2$
Average power	$P_\infty = \lim\limits_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \lvert x(t)\rvert^2 \mathrm{d}t$	$P_\infty = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x[n]\rvert^2$
Energy signal	$E_\infty < \infty$, $P_\infty = 0$	same
Power signal	$E_\infty \to \infty$, $0 < P_\infty < \infty$	same

A.2 Operations and periodicity

Operations on the time axis, and the test for periodicity.

ITEM	STATEMENT
Shift	$x(t-t_0)$: delay if $t_0 > 0$, advance if $t_0 < 0$
Reversal	$x(-t)$, $x[-n]$
Scaling	$y(t) = x(at)$: compressed if $a > 1$, stretched if $0 < a < 1$; support divided by a
Combination	$x(at-b)$: shift by b first, then scale by a
Periodicity	$x(t) = x(t+T)$; $x[n] = x[n+N]$ with N an integer
Fundamental frequency	$\omega_0 = 2\pi/T_0 = 2\pi/N_0$
Even and odd	$\mathcal{E}\mathrm{v}\{x\} = \frac{1}{2}[x(t)+x(-t)]$, $\mathcal{O}\mathrm{dd}\{x\} = \frac{1}{2}[x(t)-x(-t)]$

A.3 Impulses and steps

The unit impulse and the unit step in continuous and discrete time.

ITEM	CONTINUOUS TIME	DISCRETE TIME
Step and impulse	$\delta(t) = \frac{\mathrm{d}}{\mathrm{d}t}u(t)$, $u(t) = \int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau$	$\delta[n] = u[n] - u[n-1]$, $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$
Sampling	$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$	$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$
Sifting	$x(t_0) = \int_{-\infty}^{\infty} x(t)\delta(t-t_0)\mathrm{d}t$	$x[n_0] = \sum_n x[n]\delta[n-n_0]$
Representation	$x(t) = \int x(\tau)\delta(t-\tau)\mathrm{d}\tau$	$x[n] = \sum_k x[k]\delta[n-k]$

A.4 Complex exponentials

Complex exponentials in continuous and discrete time, and their periods.

ITEM	STATEMENT
Continuous time	$x(t) = Ce^{at}$; with $C = Ae^{j\theta}$ and $a = r + j\omega_0$: $x(t) = Ae^{rt}e^{j(\omega_0 t+\theta)}$
Period	$T_0 = 2\pi/\omega_0$, always periodic for $\omega_0 \neq 0$
Discrete time	$x[n] = C\alpha^n$ with $\alpha = e^{\beta}$; growth boundary at $\lvert\alpha\rvert = 1$
Period	$N = \frac{2\pi}{\omega_0}k$; periodic only if $\omega_0/2\pi \in \mathbb{Q}$
Frequency wrap-around	$e^{j(\omega_0+2\pi)n} = e^{j\omega_0 n}$

A.5 Systems and convolution

System properties, in general and for an LTI system in terms of h.

PROPERTY	GENERAL CRITERION	LTI CRITERION IN TERMS OF h
Memoryless	output at t uses only input at t	$h(t) = a\delta(t)$ or $h[n] = a\delta[n]$
Invertible	distinct inputs give distinct outputs	$h * g = \delta$ for some g
Causal	output uses only $\tau \leq t$	$h(t) = 0$ for $t < 0$; $h[n] = 0$ for $n < 0$
BIBO stable	bounded input gives bounded output	$\int \lvert h\rvert\, \mathrm{d}t < \infty$; $\sum_k \lvert h[k]\rvert < \infty$
Time invariant	$x(t-t_0) \to y(t-t_0)$	—
Linear	$ax_1 + bx_2 \to ay_1 + by_2$	—

CONVOLUTION

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,\mathrm{d}\tau$$

Convolution is commutative, distributive, and associative. The output support is the sum of the input supports. The total areas, or the total sums in discrete time, multiply.

A.7 Fourier series properties

A periodic signal of period T_0 with $\omega_0 = 2\pi/T_0$, or a periodic sequence of period N with $\omega_0 = 2\pi/N$. In both columns $x \leftrightarrow a_k$ and $y \leftrightarrow b_k$.

Fourier series properties in continuous and discrete time.

PROPERTY	CONTINUOUS TIME	DISCRETE TIME
Linearity	$Ax(t) + By(t) \leftrightarrow Aa_k + Bb_k$	$Ax[n] + By[n] \leftrightarrow Aa_k + Bb_k$
Time shift	$x(t - t_0) \leftrightarrow a_k e^{-jk\omega_0 t_0}$	$x[n - n_0] \leftrightarrow a_k e^{-jk(2\pi/N)n_0}$
Frequency shift	$e^{jM\omega_0 t}x(t) \leftrightarrow a_{k-M}$	$e^{jM(2\pi/N)n}x[n] \leftrightarrow a_{k-M}$
Conjugation	$x^*(t) \leftrightarrow a^*_{-k}$	$x^*[n] \leftrightarrow a^*_{-k}$
Time reversal	$x(-t) \leftrightarrow a_{-k}$	$x[-n] \leftrightarrow a_{-k}$
Time scaling	$x(\alpha t)$, $\alpha > 0$: a_k, period T_0/α	$x_{(m)}[n]$: $\frac{1}{m}a_k$, period mN
Periodic convolution	$\int_{T_0} x(\tau)y(t-\tau)\mathrm{d}\tau \leftrightarrow T_0 a_k b_k$	$\sum_{r=\langle N\rangle} x[r]y[n-r] \leftrightarrow N a_k b_k$
Multiplication	$x(t)y(t) \leftrightarrow \sum_{\ell=-\infty}^{\infty} a_\ell b_{k-\ell}$	$x[n]y[n] \leftrightarrow \sum_{\ell=\langle N\rangle} a_\ell b_{k-\ell}$
Derivative or difference	$\mathrm{d}x/\mathrm{d}t \leftrightarrow jk\omega_0 a_k$	$x[n] - x[n-1] \leftrightarrow (1 - e^{-jk(2\pi/N)})a_k$
Integral or running sum	$\int_{-\infty}^{t} x\,\mathrm{d}\tau \leftrightarrow \frac{a_k}{jk\omega_0}$, only if $a_0 = 0$	$\sum_{r=-\infty}^{n} x[r] \leftrightarrow \frac{a_k}{1-e^{-jk(2\pi/N)}}$, only if $a_0 = 0$
Real signal	$a_k = a^*_{-k}$; $\lvert a_k\rvert$ even, $\angle a_k$ odd	same
Real and even	a_k real and even	same
Real and odd	a_k purely imaginary and odd	same
Even-odd parts	$\mathcal{E}\mathrm{v}\{x\} \leftrightarrow \mathrm{Re}\{a_k\}$, $\mathcal{O}\mathrm{dd}\{x\} \leftrightarrow j\,\mathrm{Im}\{a_k\}$	same
Parseval	$\frac{1}{T_0}\int_{T_0} \lvert x\rvert^2 \mathrm{d}t = \sum_k \lvert a_k\rvert^2$	$\frac{1}{N}\sum_{n=\langle N\rangle} \lvert x\rvert^2 = \sum_{k=\langle N\rangle} \lvert a_k\rvert^2$

The two columns differ in three rows because discrete-time coefficients repeat, $a_k = a_{k+N}$. For multiplication, the discrete-time sum therefore covers one period instead of all integers. Time expansion carries the factor $\frac{1}{m}$ because the average uses m times as many samples. Discrete time also uses a first difference where continuous time uses a derivative. At $k = 0$, integration or accumulation requires zero mean; otherwise the result grows without bound.

A.8 Continuous-time Fourier transform

Properties of the continuous-time Fourier transform.

PROPERTY	STATEMENT
Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(j\omega)$
Frequency shift	$e^{j\omega_0 t} x(t) \leftrightarrow X(j(\omega - \omega_0))$
Conjugation	$x^*(t) \leftrightarrow X^*(-j\omega)$
Time reversal	$x(-t) \leftrightarrow X(-j\omega)$
Time and frequency scaling	$x(at) \leftrightarrow \frac{1}{\lvert a\rvert} X(j\omega/a)$
Convolution	$x(t) * h(t) \leftrightarrow X(j\omega)H(j\omega)$
Multiplication	$x(t)y(t) \leftrightarrow \frac{1}{2\pi} X(j\omega) * Y(j\omega)$
Differentiation in time	$\mathrm{d}^n x/\mathrm{d}t^n \leftrightarrow (j\omega)^n X(j\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\mathrm{d}\tau \leftrightarrow \frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in frequency	$t\,x(t) \leftrightarrow j\,\mathrm{d}X(j\omega)/\mathrm{d}\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$
Real signal	$X(-j\omega) = X^*(j\omega)$; $\lvert X\rvert$ even, $\angle X$ odd
Real and even	$X(j\omega)$ real and even
Real and odd	$X(j\omega)$ purely imaginary and odd
Even-odd parts	$\mathcal{E}\mathrm{v}\{x\} \leftrightarrow \mathrm{Re}\{X\}$, $\mathcal{O}\mathrm{dd}\{x\} \leftrightarrow j\,\mathrm{Im}\{X\}$
Parseval	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \mathrm{d}t = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2 \mathrm{d}\omega$

Continuous-time Fourier transform pairs.

SIGNAL	TRANSFORM
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$e^{-at}u(t)$, $a>0$	$\frac{1}{a+j\omega}$
$te^{-at}u(t)$, $a>0$	$\frac{1}{(a+j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $a>0$	$\frac{1}{(a+j\omega)^n}$
$e^{-a\lvert t\rvert}$, $a>0$	$\frac{2a}{a^2+\omega^2}$
1 on $\lvert t\rvert<T_1$, 0 beyond	$\frac{2\sin(\omega T_1)}{\omega}=2T_1\,\mathrm{sinc}(\omega T_1)$
$\frac{\sin(Wt)}{\pi t}=\frac{W}{\pi}\,\mathrm{sinc}(Wt)$	1 on $\lvert\omega\rvert<W$, 0 beyond
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
$\sin\omega_0 t$	$\frac{\pi}{j}\delta(\omega-\omega_0)-\frac{\pi}{j}\delta(\omega+\omega_0)$
$\sum_k a_k e^{jk\omega_0 t}$	$\sum_k 2\pi a_k\delta(\omega-k\omega_0)$
Square wave: 1 on $\lvert t\rvert<T_1$, 0 on $T_1<\lvert t\rvert\le T/2$	$\sum_k \frac{2\sin(k\omega_0 T_1)}{k}\delta(\omega-k\omega_0)$
$\sum_k \delta(t-kT)$	$\frac{2\pi}{T}\sum_k \delta(\omega-\frac{2\pi k}{T})$

This table uses the unnormalised definition $\mathrm{sinc}(\theta)=\sin\theta/\theta$. The normalised convention divides the argument by π. Convert the argument when moving between conventions; otherwise the result loses a factor of π. The exponential pairs require $a>0$.

A.9 Discrete-time Fourier transform

Properties of the discrete-time Fourier transform.

PROPERTY	STATEMENT
Linearity	$ax_1[n] + bx_2[n] \leftrightarrow aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
Time shift	$x[n-n_0] \leftrightarrow e^{-j\omega n_0}X(e^{j\omega})$
Frequency shift	$e^{j\omega_0 n}x[n] \leftrightarrow X(e^{j(\omega-\omega_0)})$
Conjugation	$x^*[n] \leftrightarrow X^*(e^{-j\omega})$
Time reversal	$x[-n] \leftrightarrow X(e^{-j\omega})$
Time expansion	$x_{(k)}[n] \leftrightarrow X(e^{jk\omega})$
Convolution	$x[n] * h[n] \leftrightarrow X(e^{j\omega})H(e^{j\omega})$
Multiplication	$x[n]y[n] \leftrightarrow \frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})\mathrm{d}\theta$
Differencing in time	$x[n] - x[n-1] \leftrightarrow (1-e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{m=-\infty}^{n} x[m] \leftrightarrow \frac{X(e^{j\omega})}{1-e^{-j\omega}} + \pi X(e^{j0})\sum_k \delta(\omega-2\pi k)$
Differentiation in frequency	$n\,x[n] \leftrightarrow j\,\mathrm{d}X(e^{j\omega})/\mathrm{d}\omega$
Real sequence	$X(e^{-j\omega}) = X^*(e^{j\omega})$; $\lvert X\rvert$ even, $\angle X$ odd
Real and even	$X(e^{j\omega})$ real and even
Real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-odd parts	$\mathcal{E}\mathrm{v}\{x\} \leftrightarrow \mathrm{Re}\{X\}$, $\mathcal{O}\mathrm{dd}\{x\} \leftrightarrow j\,\mathrm{Im}\{X\}$
Parseval	$\sum_n \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\omega})\rvert^2 \mathrm{d}\omega$

Discrete-time Fourier transform pairs.

SEQUENCE	TRANSFORM
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi \sum_k \delta(\omega - 2\pi k)$
$a^n u[n]$, $\lvert a\rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n]$, $\lvert a\rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n]$, $\lvert a\rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^r}$
$a^{\lvert n\rvert}$, $\lvert a\rvert < 1$	$\frac{1-a^2}{1-2a\cos\omega+a^2}$
1 on $\lvert n\rvert \le N_1$, 0 beyond	$\frac{\sin(\omega(N_1+\frac{1}{2}))}{\sin(\omega/2)}$
$\frac{\sin(Wn)}{\pi n}$, $0 < W < \pi$	1 on $\lvert\omega\rvert \le W$, 0 on $W < \lvert\omega\rvert \le \pi$
1	$2\pi \sum_k \delta(\omega - 2\pi k)$
$e^{j\omega_0 n}$	$2\pi \sum_k \delta(\omega - \omega_0 - 2\pi k)$
$\cos\omega_0 n$	$\pi \sum_k [\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)]$
$\sin\omega_0 n$	$\frac{\pi}{j} \sum_k [\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)]$
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_k a_k \delta(\omega - \frac{2\pi k}{N})$
$\sum_k \delta[n-kN]$	$\frac{2\pi}{N} \sum_k \delta(\omega - \frac{2\pi k}{N})$

Every transform in this section is periodic in ω with period 2π because the time index is discrete. A periodic sequence has discrete spectral coefficients, while an aperiodic sequence has a continuous spectrum. In each case, discreteness in one domain produces periodicity in the other.

A.10 Symbols

Symbols and their meanings.

SYMBOL	MEANING
$x(t)$, $x[n]$	input signal, continuous and discrete time
$y(t)$, $y[n]$	output signal
$h(t)$, $h[n]$	impulse response of a linear time-invariant system
$\delta(t)$, $\delta[n]$	unit impulse
$u(t)$, $u[n]$	unit step
E_∞, P_∞	total energy and average power over all time
T_0, N_0	fundamental period
ω_0	fundamental angular frequency, rad/s or rad/sample
$*$	convolution
j	imaginary unit, $j^2 = -1$

PART

3

Notation

Every symbol the course defines, with the chapter that defines it. A symbol is never reused for a second meaning.

SYMBOL	MEANING
$x(t)$	Continuous-time signal or system input; $t \in \mathbb{R}$.
$x[n]$	Discrete-time signal or system input; $n \in \mathbb{Z}$ (integer time index).
$y(t),\ y[n]$	System output in continuous and discrete time.
$h(t),\ h[n]$	Impulse response: the output of an LTI system when the input is a unit impulse.
$\delta(t)$	Continuous-time unit impulse (Dirac delta). It is not an ordinary function. It is defined by its sifting action.
$\delta[n]$	Discrete-time unit impulse: 1 at $n = 0$, zero elsewhere. An ordinary sequence.
$u(t)$	Continuous-time unit step: 1 for $t \geq 0$, 0 otherwise.
$u[n]$	Discrete-time unit step: 1 for $n \geq 0$, 0 otherwise.
E_∞	Total energy over an infinite interval ($R = 1$ normalisation).
P_∞	Time-averaged power over an infinite interval.
T_0	Fundamental period of a continuous-time periodic signal: the smallest $T > 0$ with $x(t) = x(t + T)$.
N_0	Fundamental period of a discrete-time periodic signal: the smallest integer $N > 0$ with $x[n] = x[n + N]$.
ω_0	Fundamental angular frequency: $2\pi/T_0$ (rad/s) or $2\pi/N_0$ (rad/sample).
$*$	Convolution operator. Defined only for linear time-invariant systems.
$C,\ a,\ \alpha,\ \beta$	Complex exponential parameters: $x(t) = Ce^{at}$, $x[n] = Ce^{\beta n} = C\alpha^n$ with $\alpha = e^\beta$.
$\mathcal{E}\text{v}\{\cdot\},\ \mathcal{O}\text{dd}\{\cdot\}$	Even and odd parts: $\mathcal{E}\text{v}\{x\} = \frac{1}{2}[x(t) + x(-t)]$ and $\mathcal{O}\text{dd}\{x\} = \frac{1}{2}[x(t) - x(-t)]$.
S	System operator mapping an input signal to an output signal.

Signals and Systems · Formula and Notation Reference

VERSION	DATE	CHANGES
v0	23 September 2026	First edition.